DATA 442: Neural Networks & Deep Learning

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icss.wm.edu/data442/

All Colors Activation

45
65
78
97

Blue Filter Activation

12
78
9
6

Green Filter Activation

4
5
8
78

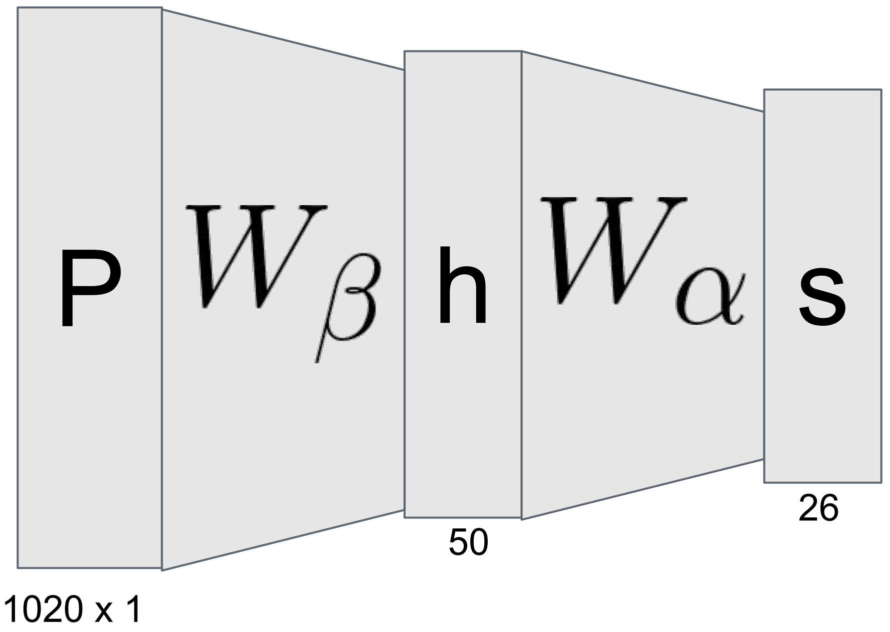
Red Filter Activation

3
12
8
1

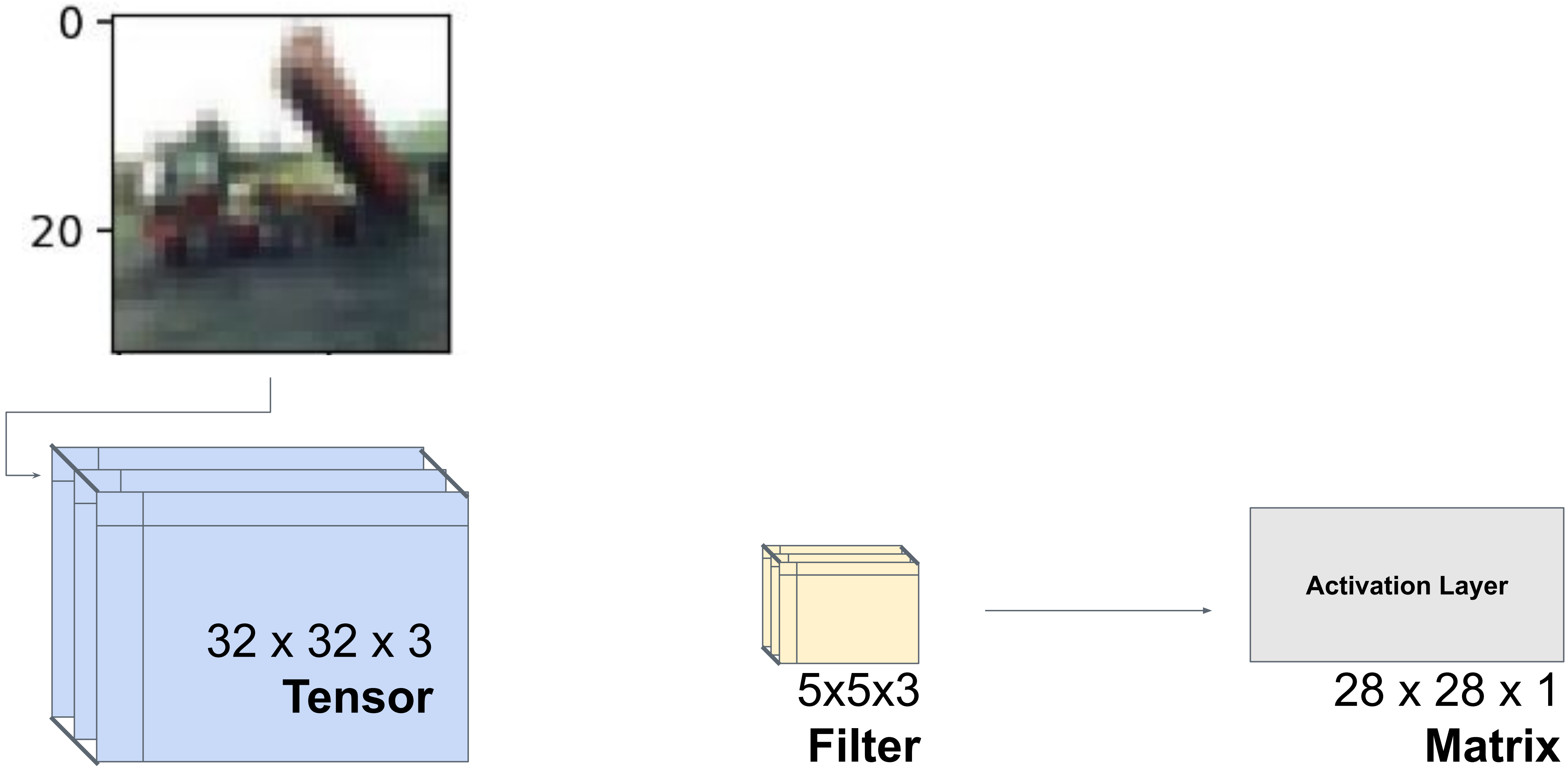
...

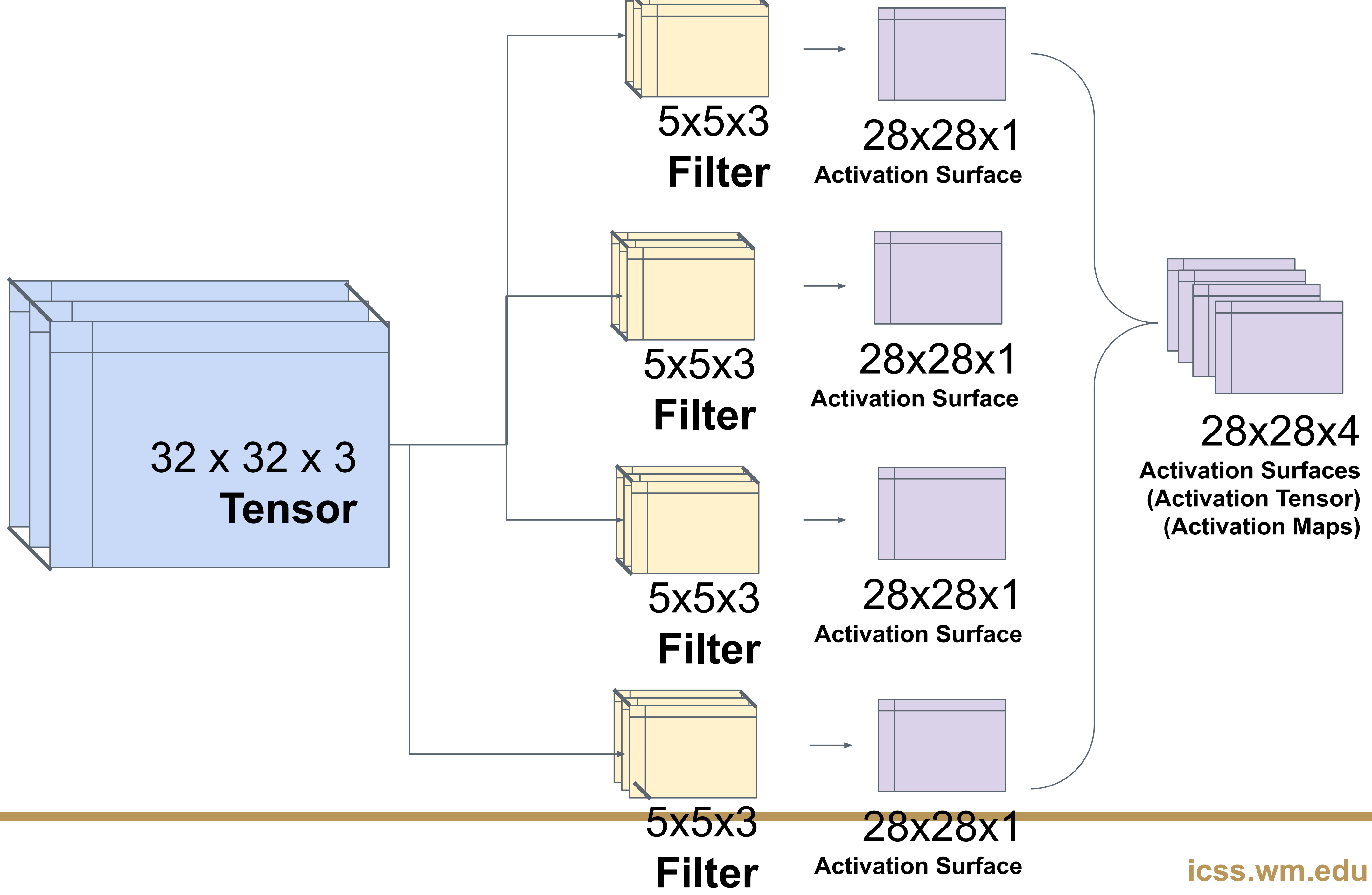
Filter 255 Activation

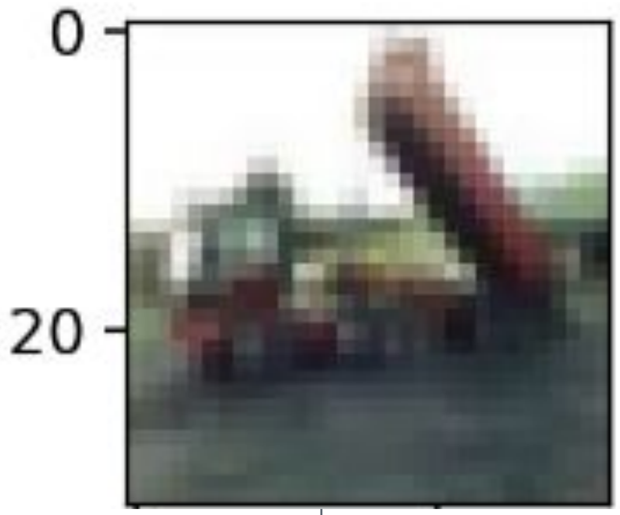
3
12
8
1



- Four Choices (Hyperparameters)
  - Number of Filters
  - Filter Dimensions
  - Stride
  - Zero Padding


0
20
32 x 32 x 3
Tensor
5x5x3
Filter
Activation Layer
28 x 28 x 1
Matrix


32 x 32 x 3
Tensor
5x5x3
Filter
28x28x1
Activation Surface
5x5x3
Filter
28x28x1
Activation Surface
5x5x3
Filter
28x28x1
Activation Surface
5x5x3
Filter
28x28x1
Activation Surface
28x28x4
Activation Surfaces
(Activation Tensor)
(Activation Maps)



Convolution (5x5x3 Filter)

Activation Layer (28 x 28 x 1 Matrix)

Input Activation Array 1

784

●

Weights for each Pixel, for each of 10 CIFAR classes

784

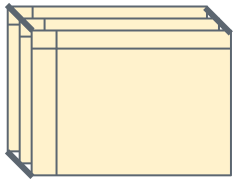
10

=

Output Scores Array

10

1



5x5x3
Filter

1	5	1	8	6
9	8	5	4	3
1	6	4	3	1
1	5	8	3	1
1	2	2	1	1

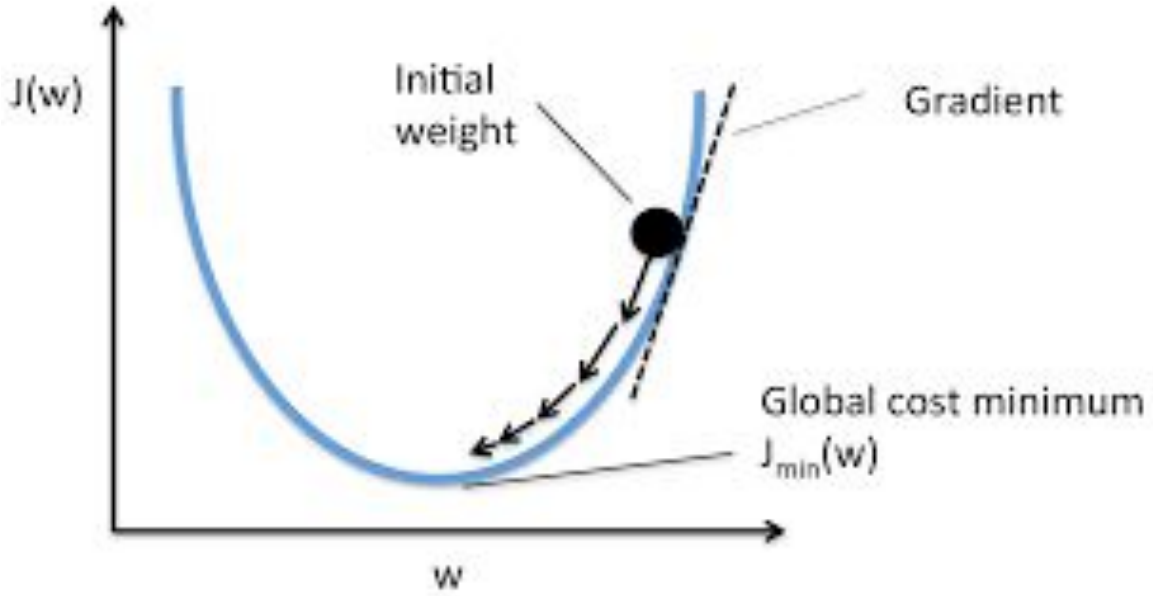
5	8	4	2	0
6	6	4	5	1
9	8	9	1	1
7	6	5	0	3
5	1	1	0	2

5	6	8	7	7
2	1	1	1	4
2	5	2	5	5
3	5	2	3	5
4	5	2	3	5

Optimization

Goal: Find the best weights parameters to minimize a loss function.

Approaches we've discussed: Gradient Descent, Stochastic Gradient Descent, Mini-batch SGD.



Optimization

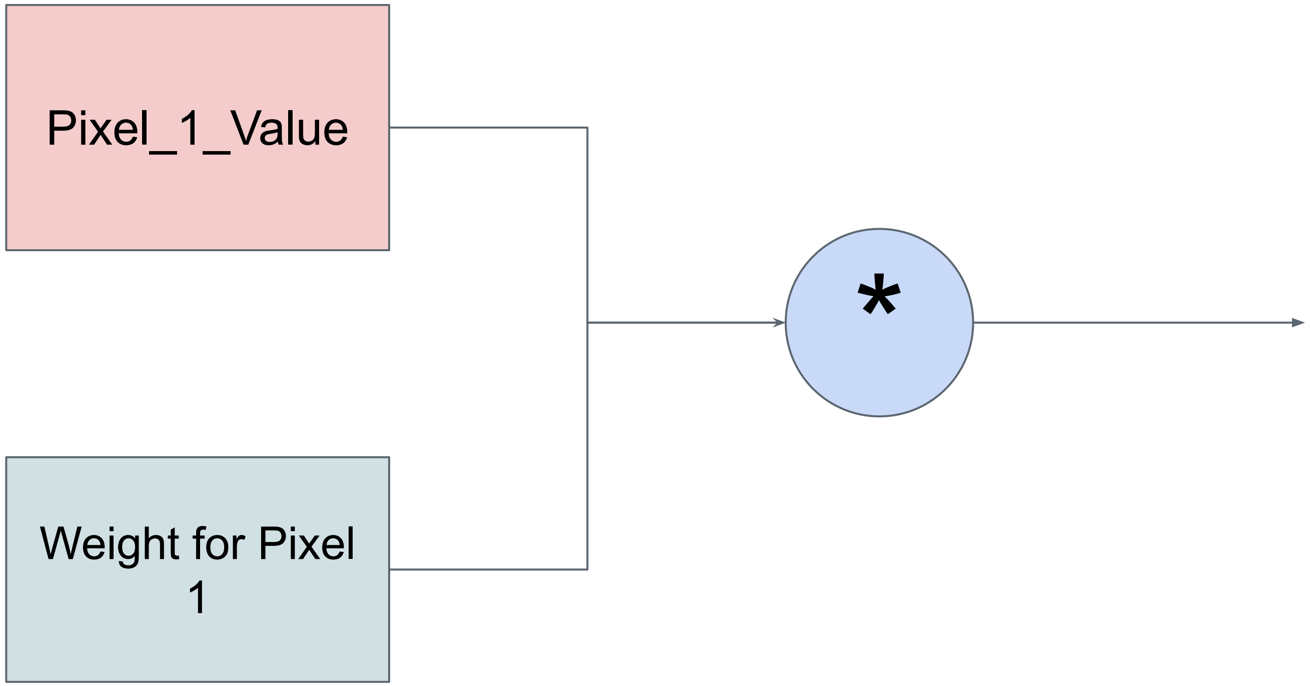
Example (Mini-batch SGD):

1. Sample your data (batch size)
2. Run a forward propagation through your network.
3. Calculate your loss
4. Backpropogate to calculate gradients of weights with respect to loss.
5. Update weights using the gradient.
6. Repeat until some threshold is reached (i.e., number of iterations).

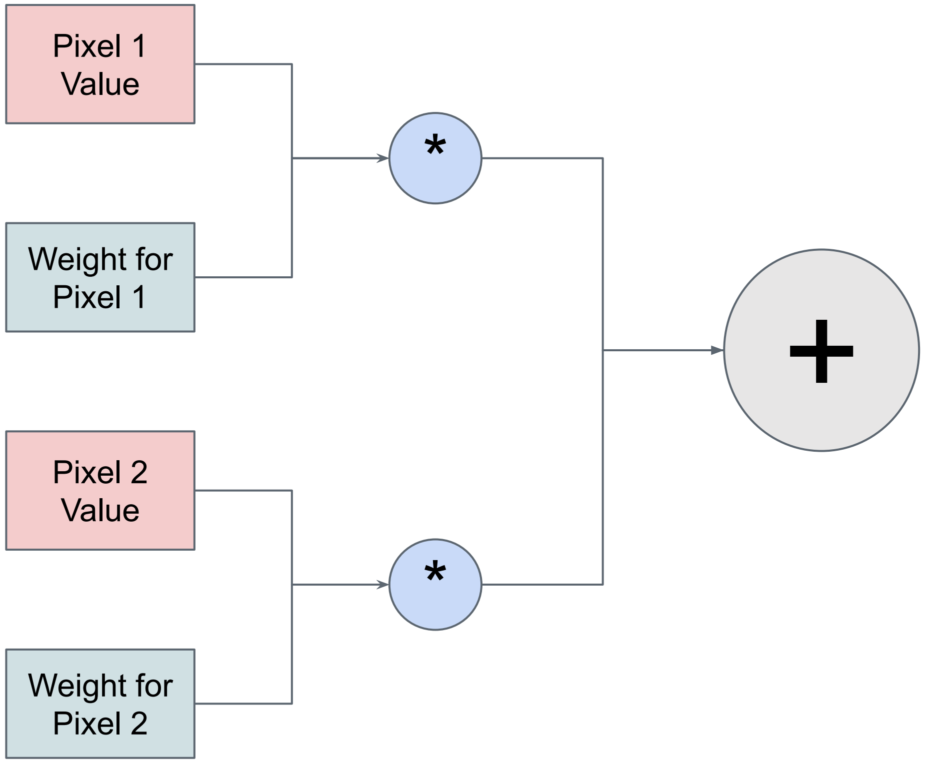
Building and Optimizing a Neural Network

- Define Network Architecture (Computational Graph)
- Train / Optimize the Network
- Evaluation

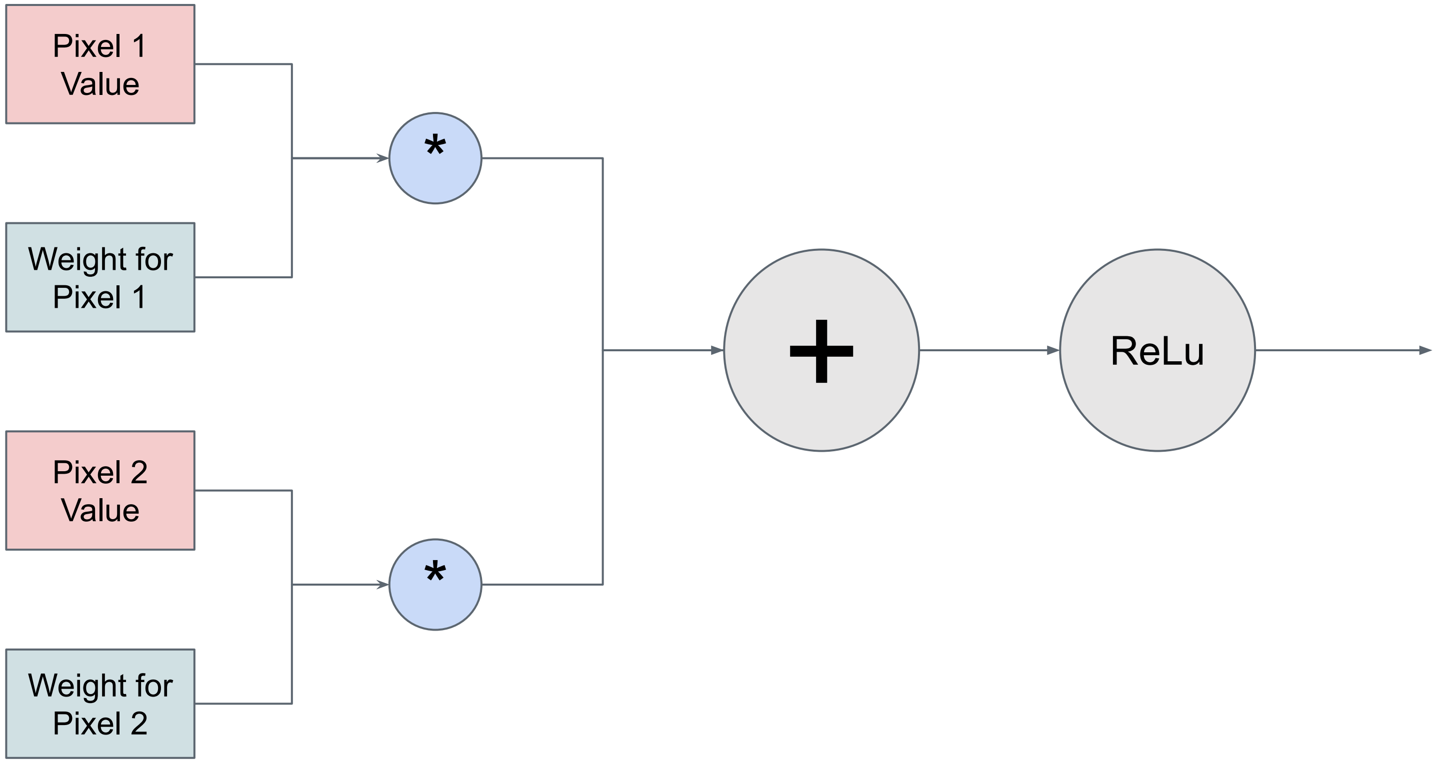
Network Architecture: Fundamentals



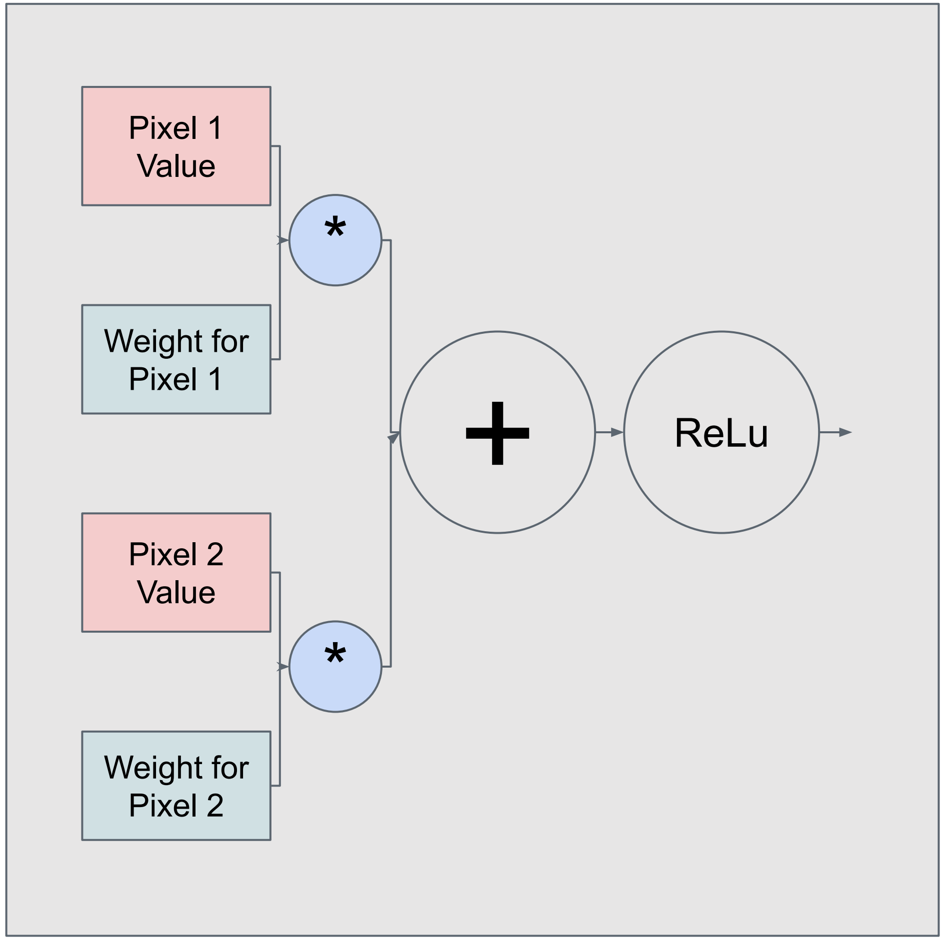
Network Architecture: Fundamentals

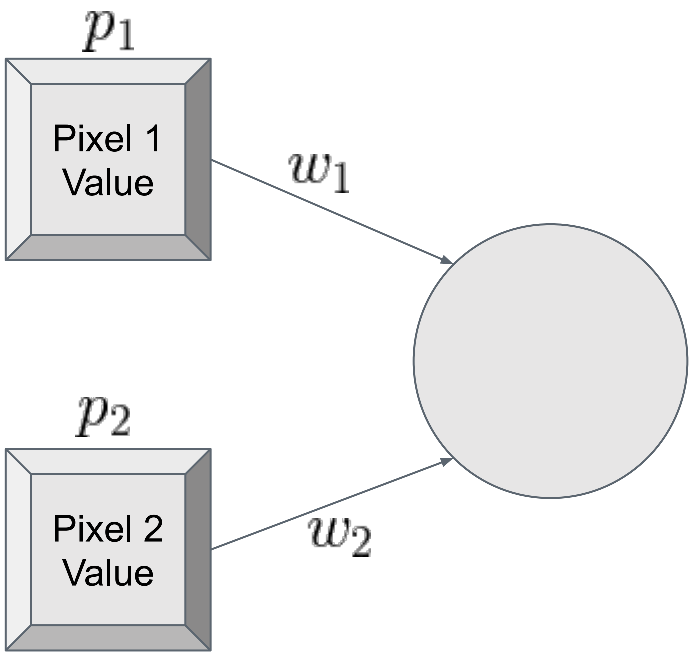


Network Architecture: Fundamentals

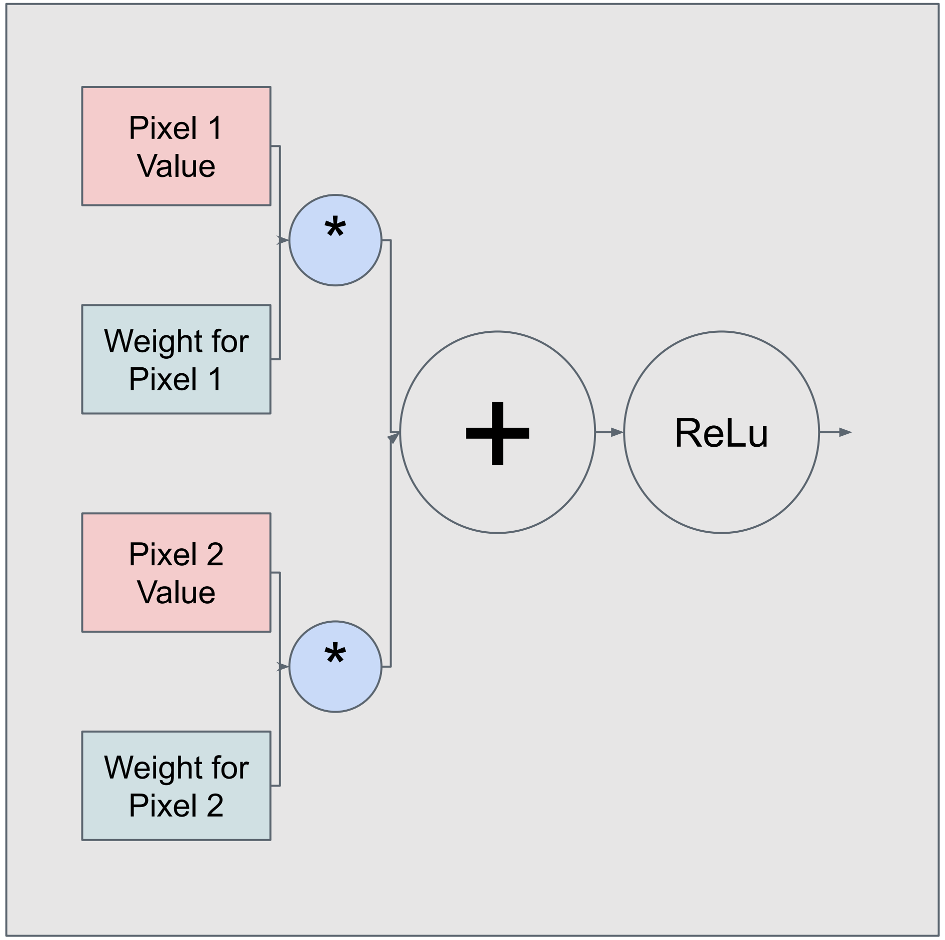


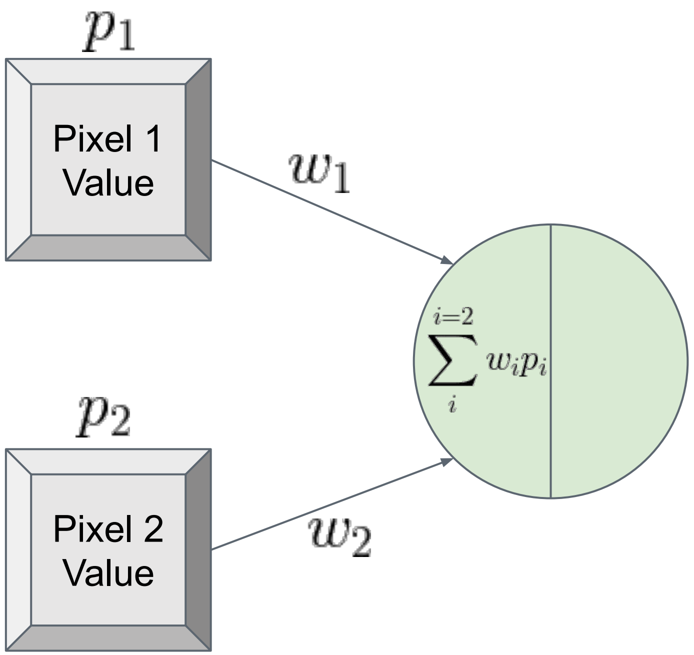
Network Architecture: Fundamentals



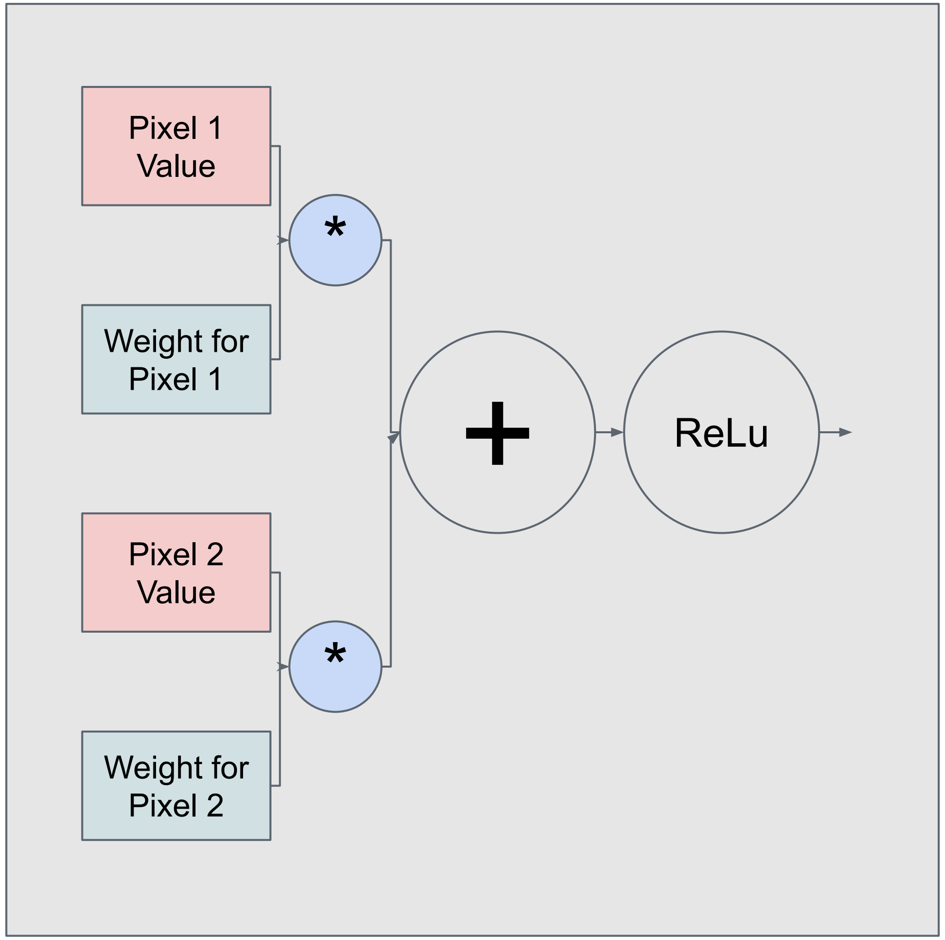


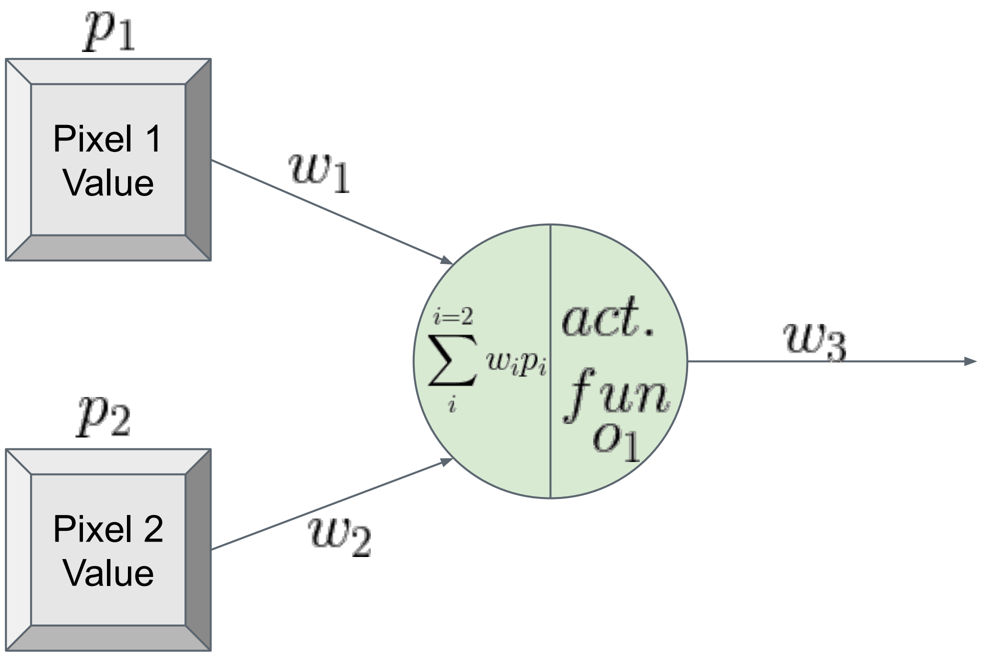
Network Architecture: Fundamentals



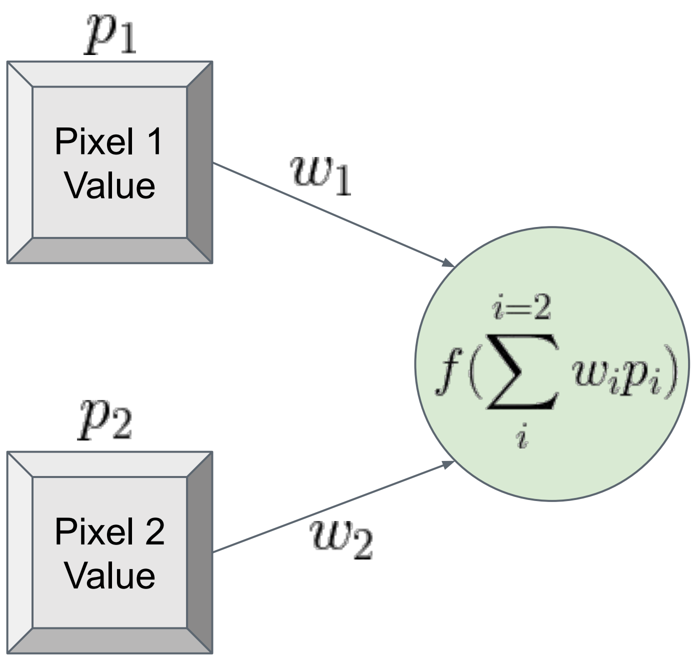


Network Architecture: Fundamentals





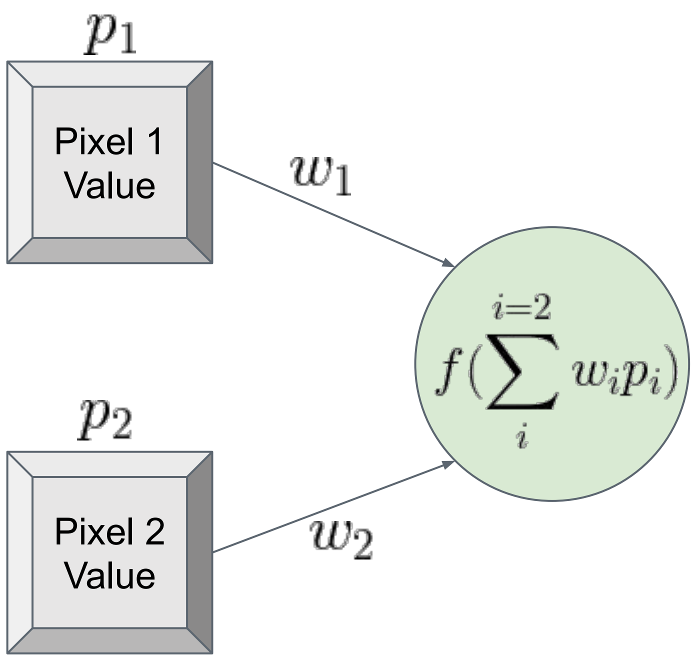
Network Architecture: Activation Function

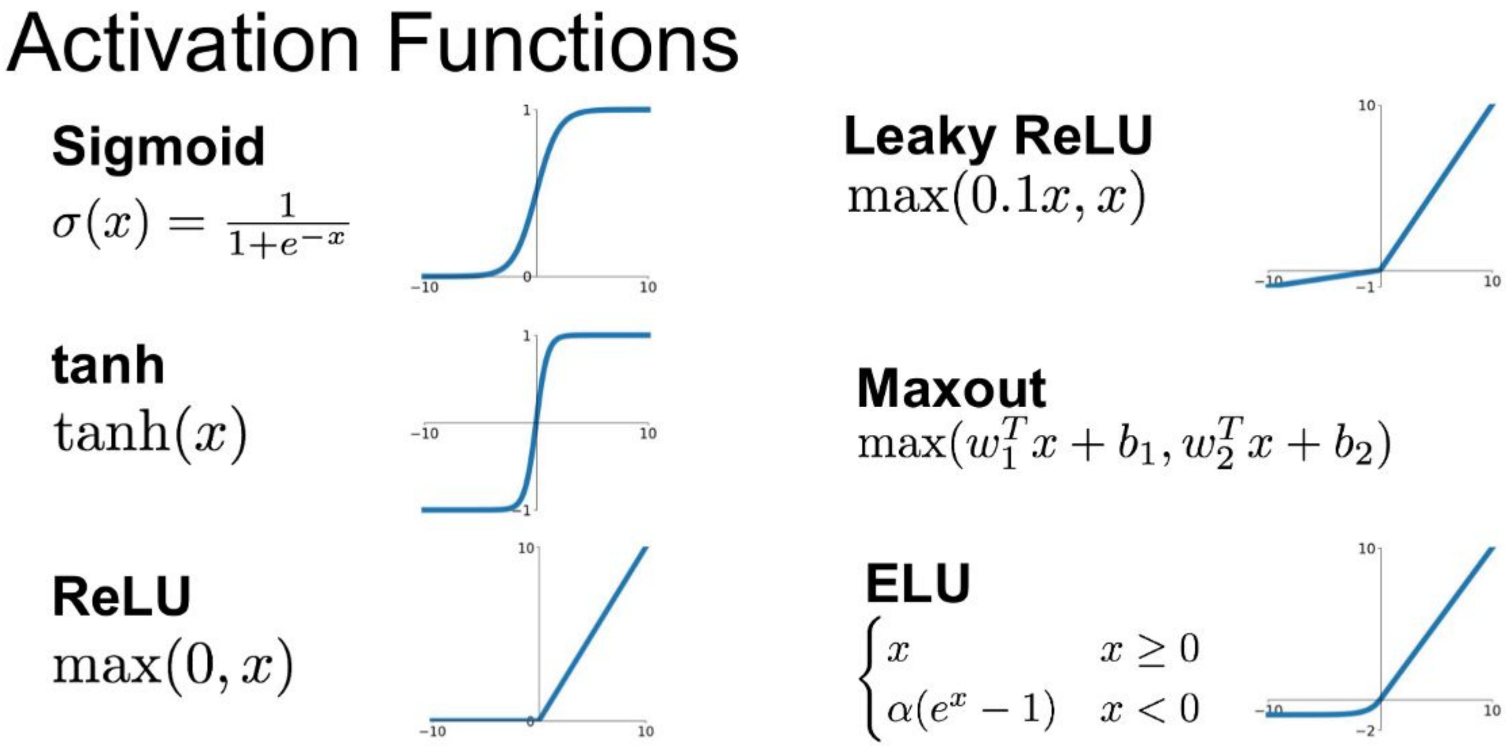


$$f(\sum_{i}^{i=2} w_i p_i)$$

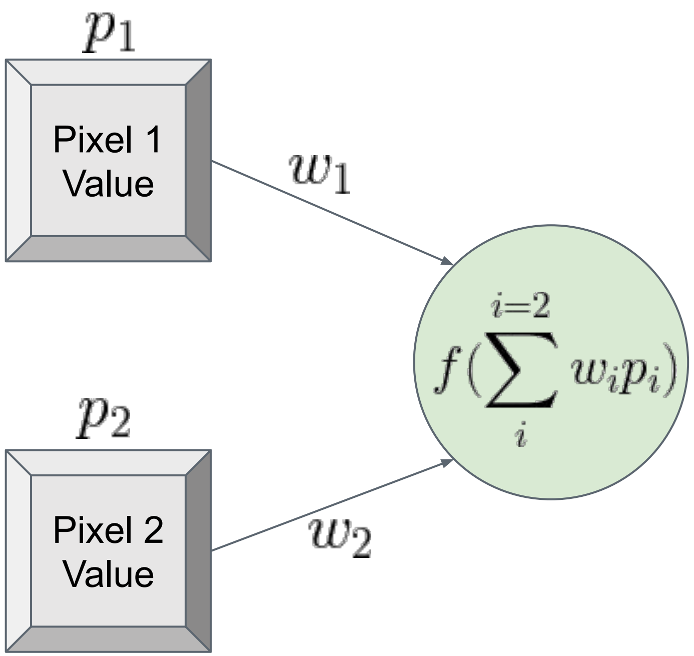
```
def activationFunction(input):
    return(input * 1)
```

Network Architecture: Activation Function

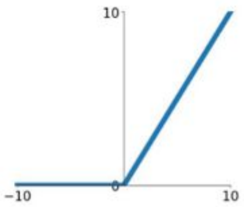




Network Architecture: Activation Function

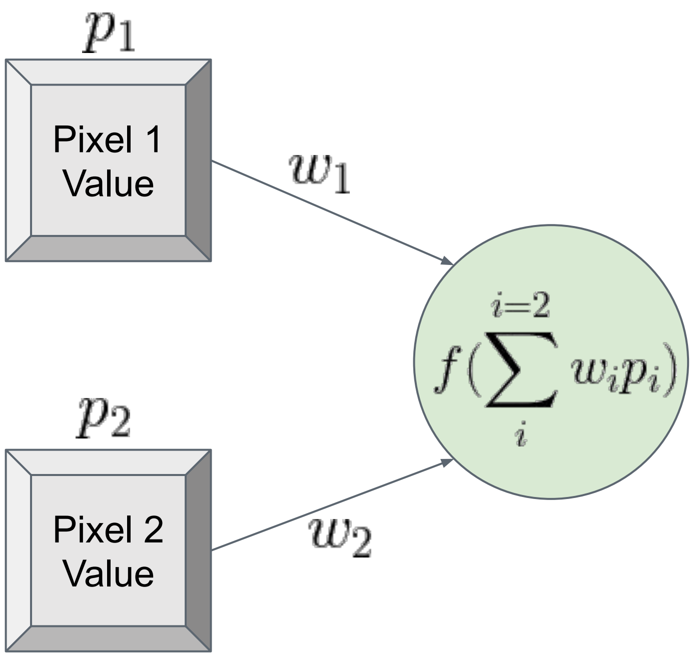


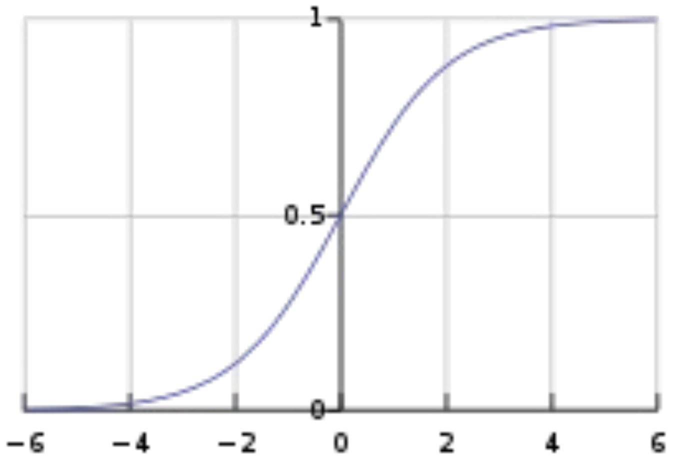
ReLU
$\max(0, x)$




```
def activationFunction(input):
    return(max(input,0))
```

Sigmoid Activation Function



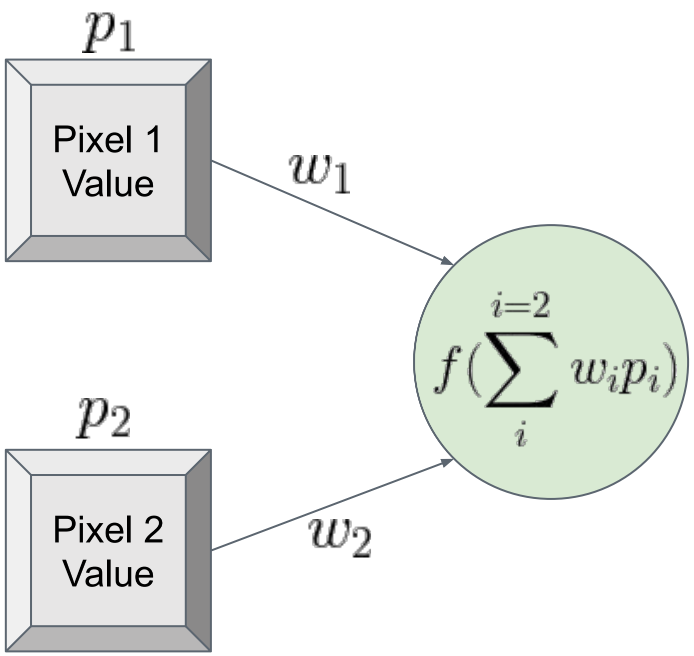


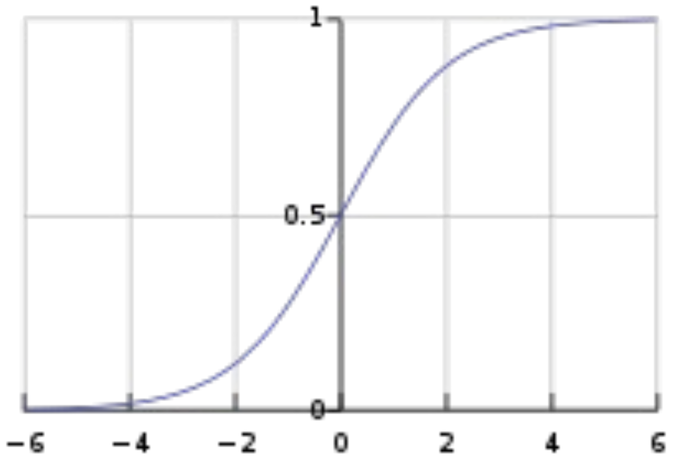
$$\sigma(x) = \frac{1}{(1 + e^{-x})}$$

Features

- Output of function falls between 0 and 1.
- Roughly approximates how a neuron works - 0 values until some threshold is reached, then 1.

Sigmoid Activation Function





$$\sigma(x) = \frac{1}{(1 + e^{-x})}$$

Features

- Output of function falls between 0 and 1.
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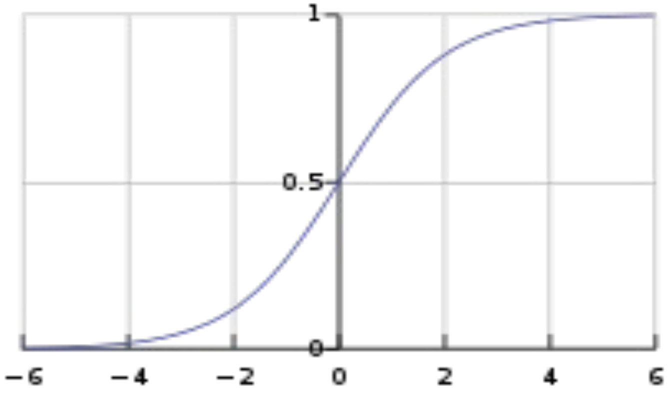
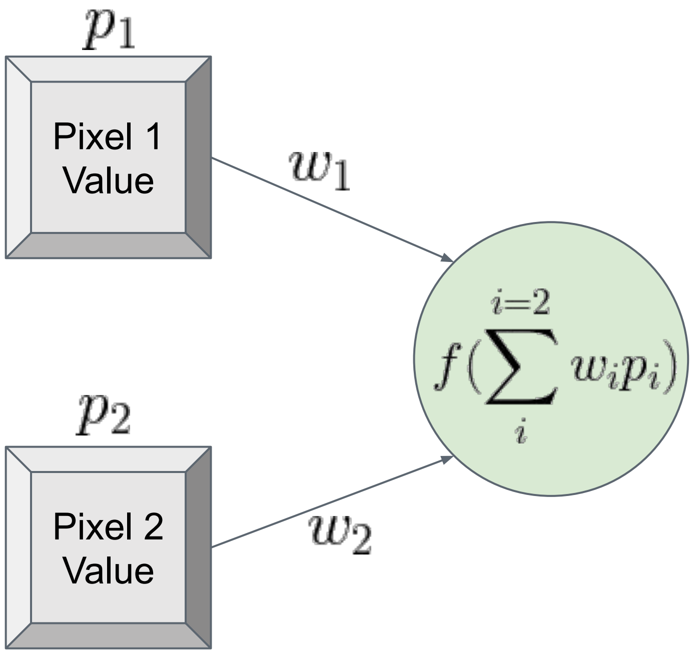
Challenges

- Gradient Decay & Saturation
- Not Zero-Centered & Unidirectional Gradient Solutions

Sigmoid Activation Function

$$\sigma(x) = \frac{1}{(1 + e^{-x})}$$





Consider if pixel 1 value and pixel 2 value are both 1, and both weights are 10.

What would a change in the weight of -1 do to the sigmoid activation function?

Features

- Output of function falls between 0 and 1.
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Challenges

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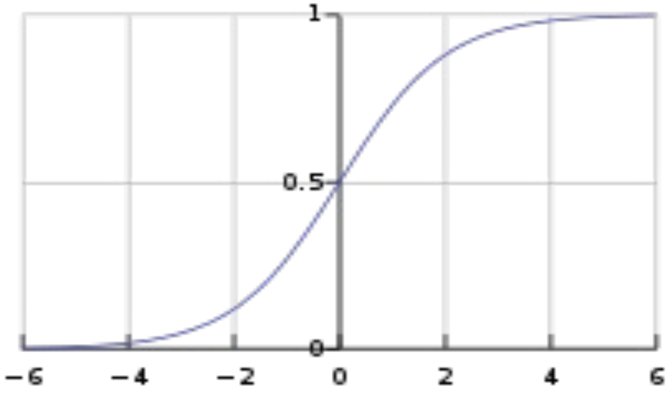
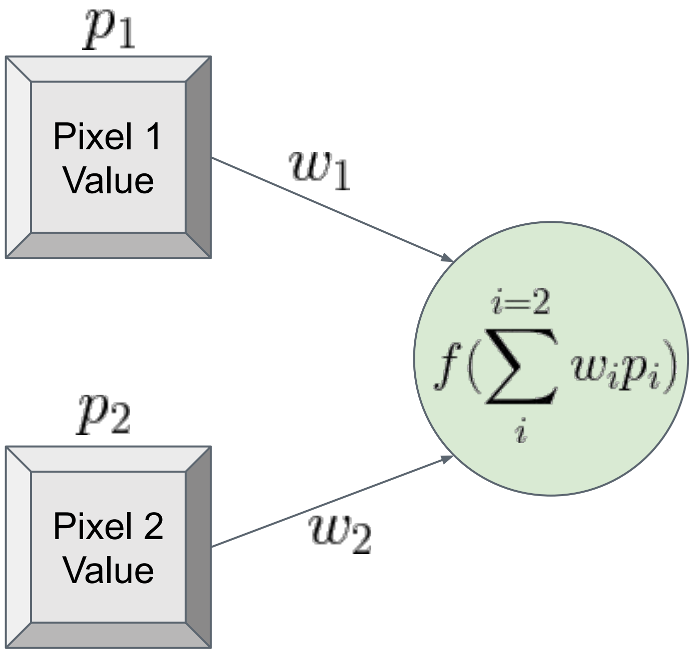
$$\sigma(x) = \frac{1}{(1 + e^{-x})}$$





Consider if pixel 1 value and pixel 2 value are both 1, and both weights are 10.

What would a change in the weight of -1 do to the sigmoid activation function?

Nothing (the gradient would be 0)

Features

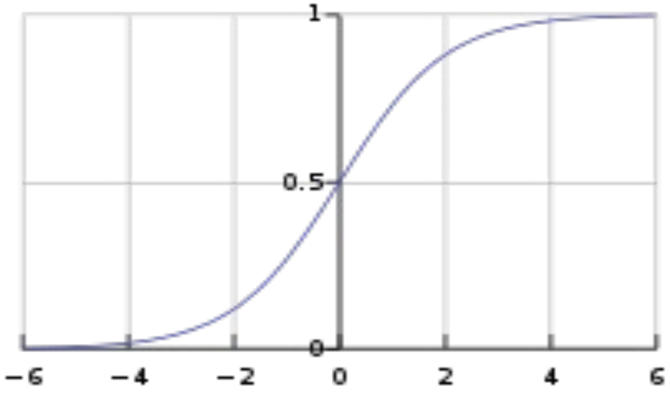
- Output of function falls between 0 and 1.
- Roughly approximates how a neuron works - 0 values until some threshold is reached, then 1.

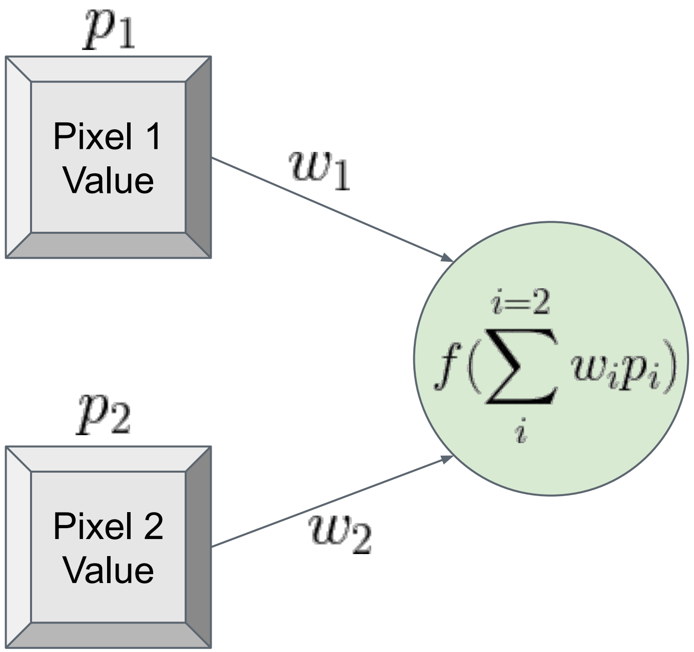
Challenges

- Gradient Decay & Saturation
- Not Zero-Centered & Unidirectional Gradient Solutions

Sigmoid Activation Function

$$\sigma(x) = \frac{1}{(1 + e^{-x})}$$





Now consider the directionality of the gradient. If all of your inputs into a given neuron are positive, then gradients will all always be positive or negative - no mixing of positive and negative gradients during back propagation.

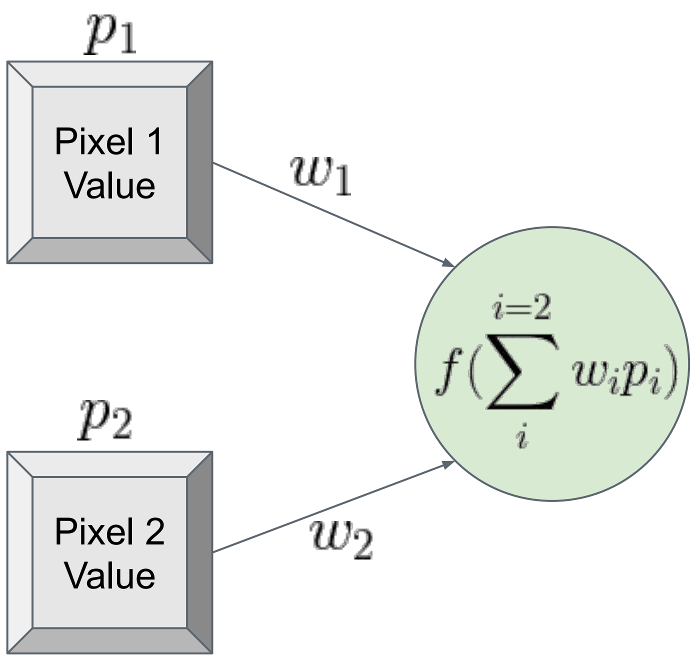
Features

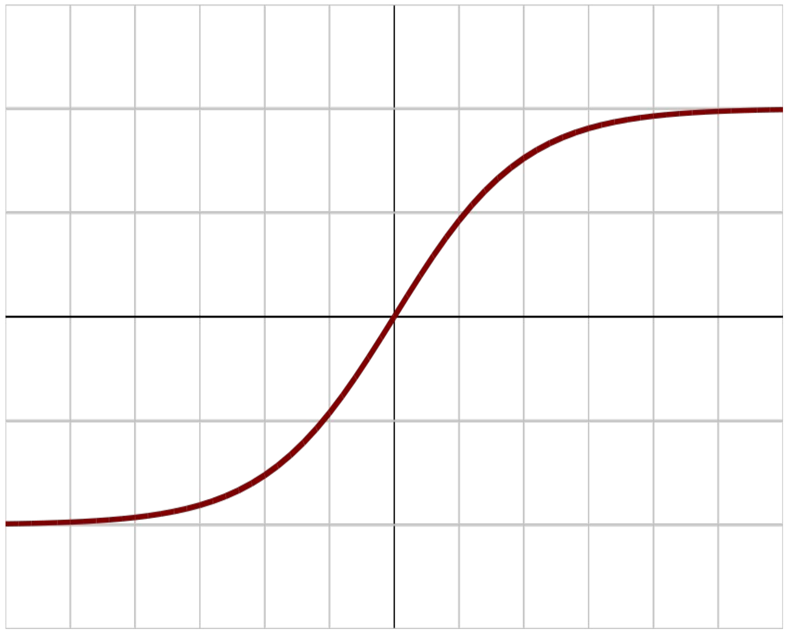
- Output of function falls between 0 and 1.
- Roughly approximates how a neuron works - 0 values until some threshold is reached, then 1.

Challenges

- Gradient Decay & Saturation
- Not Zero-Centered & Unidirectional Gradient Solutions

tanh Activation Function





Features

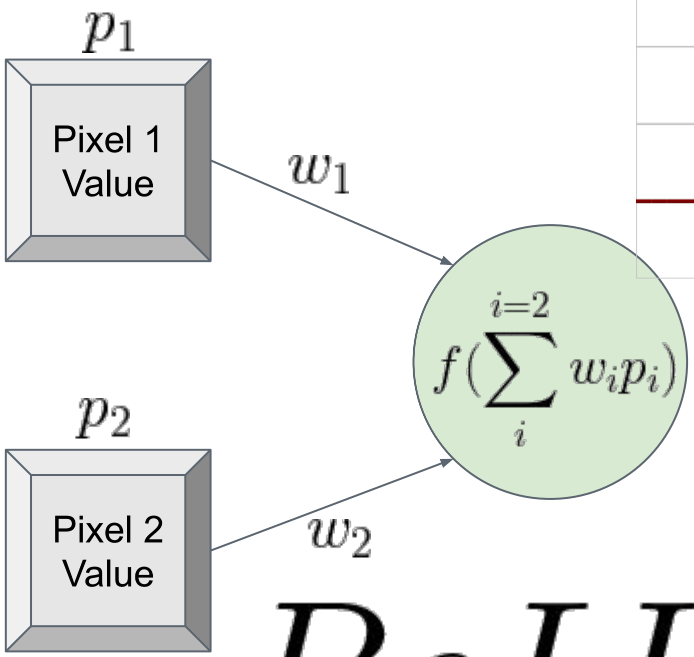
- Output of function falls between -1 and 1.
- Roughly approximates how a neuron works - 0 values until some threshold is reached, then 1.
- Zero Centered

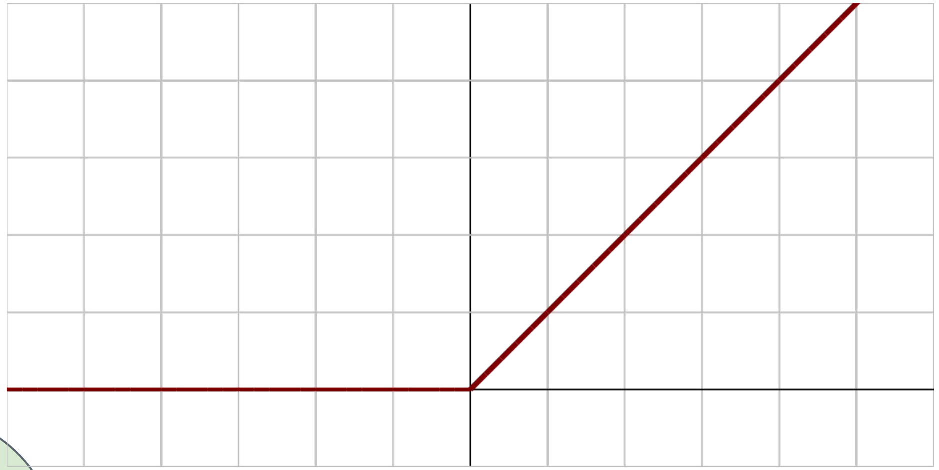
Challenges

- Gradient Decay & Saturation

$$tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Rectified Linear Unit (ReLU) Activation Function





Features

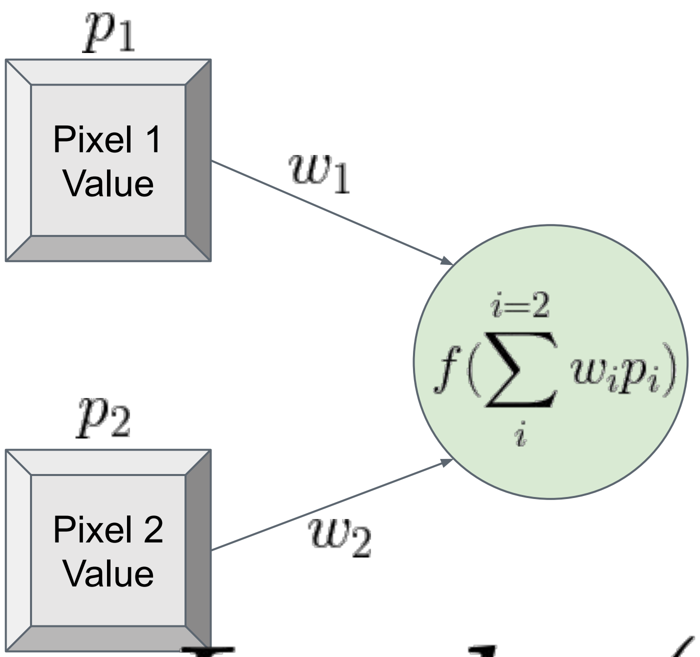
- No saturation in positive direction
- Very, very simple (and, thus, computationally efficient)
- Roughly approximates how a neuron works - 0 values until 0 is reached, then x.

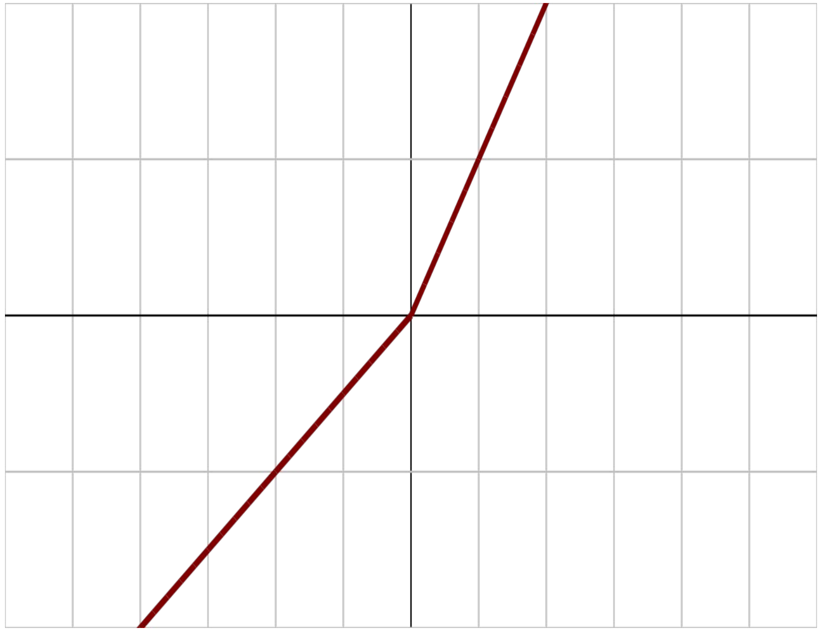
Challenges

- Not zero-centered
- Gradient Decay / Saturation if X < 0

$$ReLU(x) = max\{0, X\}$$

Leaky ReLU Activation Function





Features

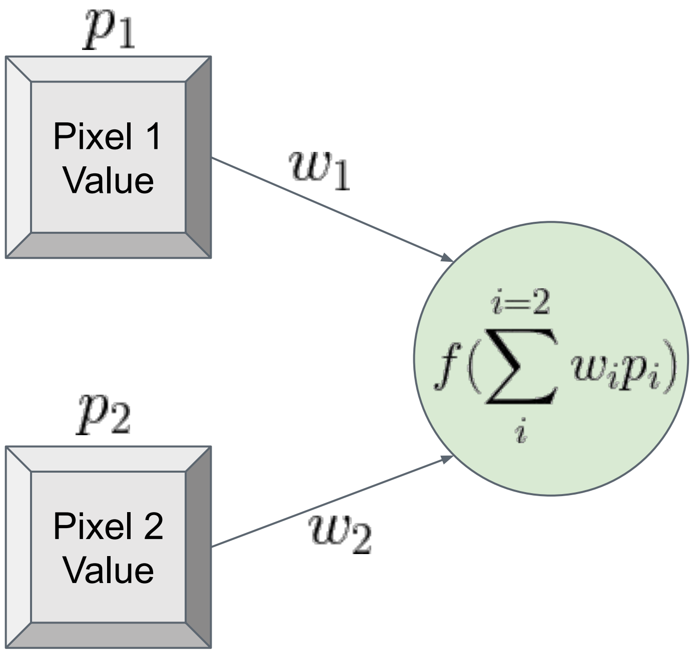
- No saturation (and, thus, no ReLU “death”).
- Still very simple (and, thus, computationally efficient)
- Roughly approximates how a neuron works - small values until 0 is reached, then x.

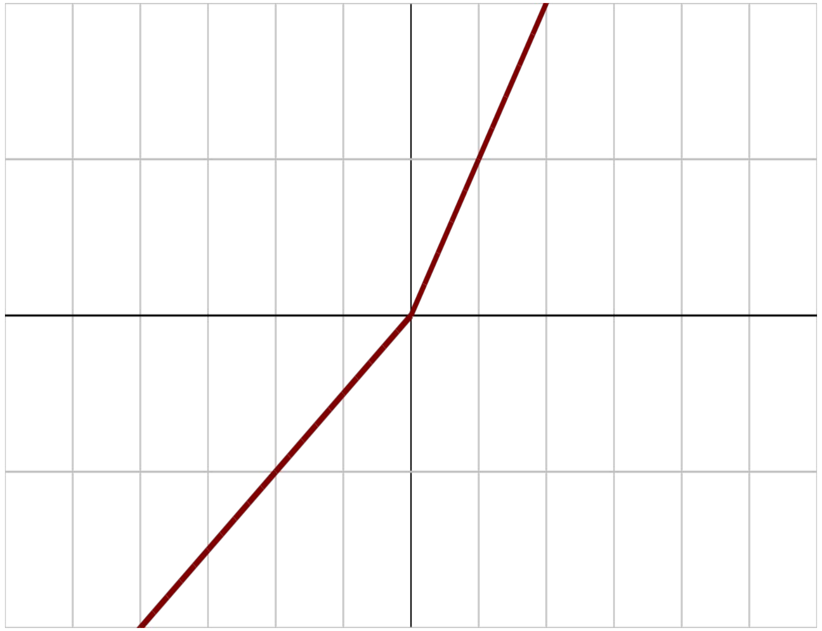
Challenges

- Not zero-centered

$$Leaky(x) = max\{0.01X, X\}$$

Parametric ReLU Activation Function





Features

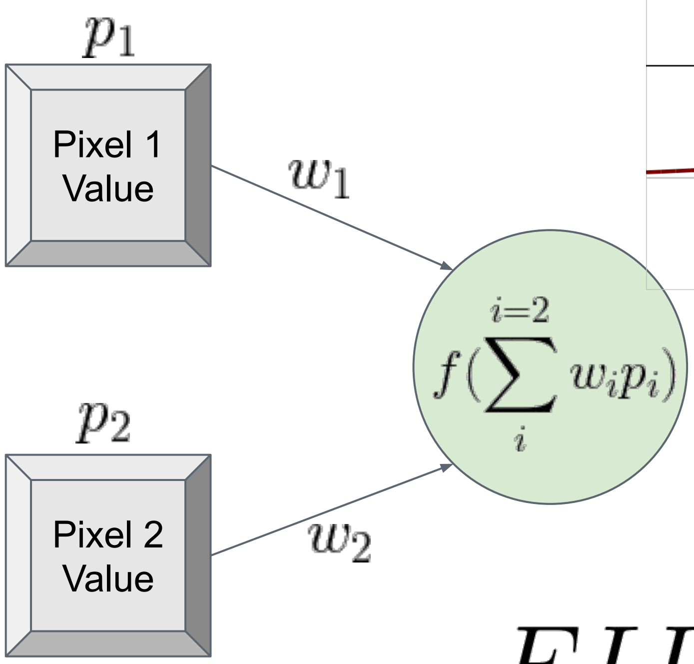
- No saturation (and, thus, no ReLU “death”).
- Still very simple (and, thus, computationally efficient)
- Roughly approximates how a neuron works - small values until 0 is reached, then x.
- Parameterized, and can be fit during optimization.

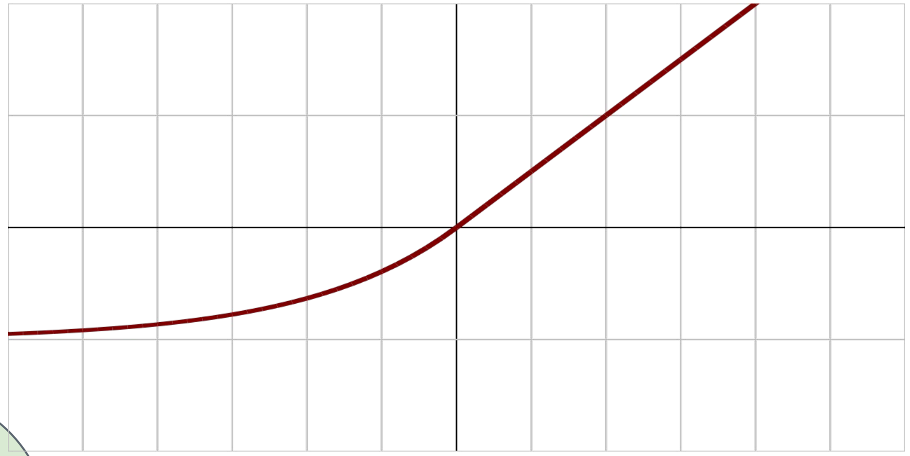
Challenges

- Not zero-centered

$$PReLU(x) = max\{\alpha * X, X\}$$

Exponential Linear Units (ELU) Activation Function





Features

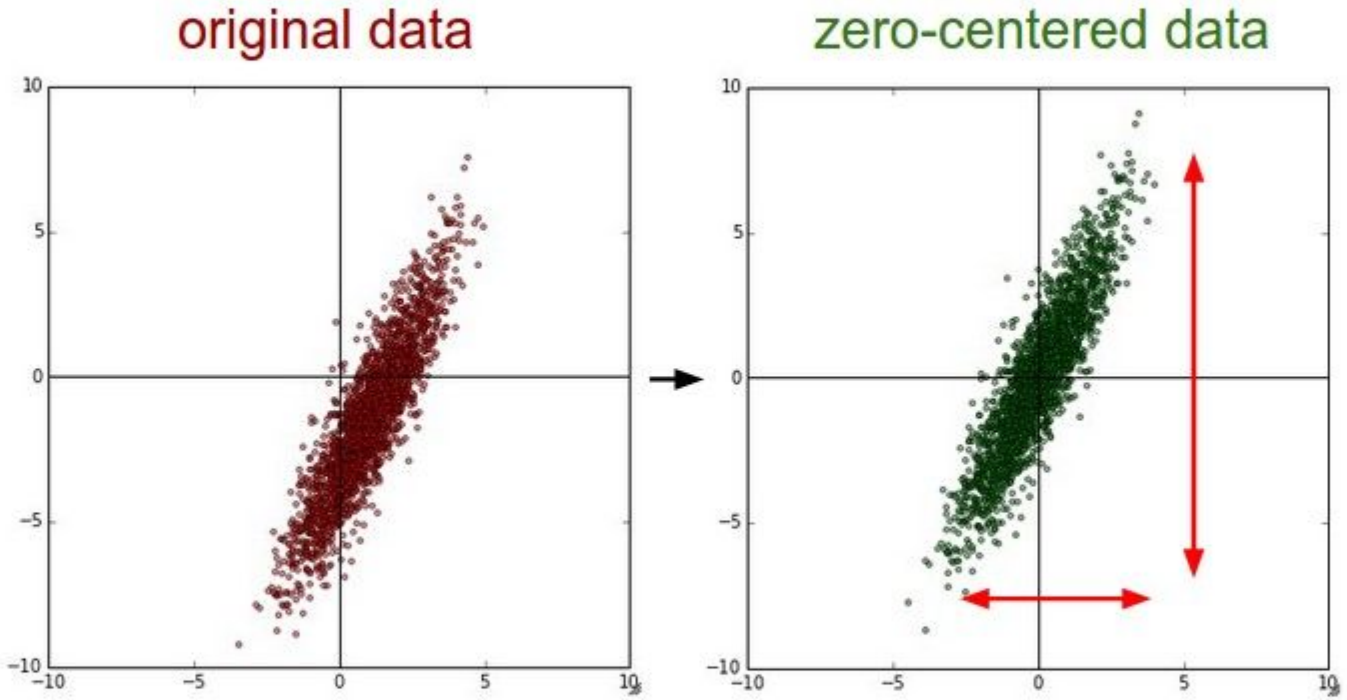
- No saturation if x > 0
- Roughly approximates how a neuron works - small values until 0 is reached, then x.
- Parameterized, and can be fit during optimization.
- Close to mean centered.

Challenges

- Potential for saturation if x < 0.
- Not actually zero-centered, though it is much closer.

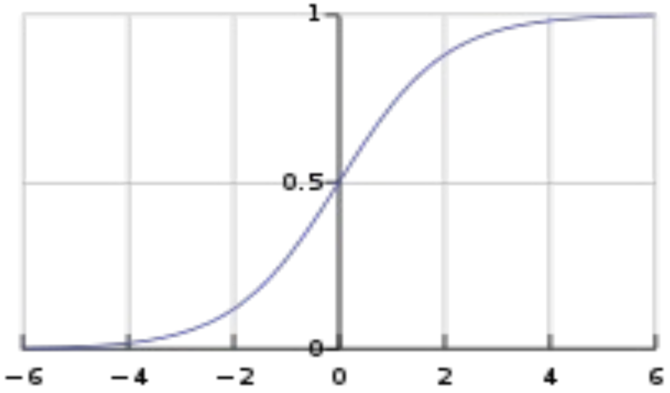
$$ELU(x) = \begin{cases} x & \text{if x} > 0 \\ \alpha(exp(x) - 1) & \text{if x} \leq 0 \end{cases}$$

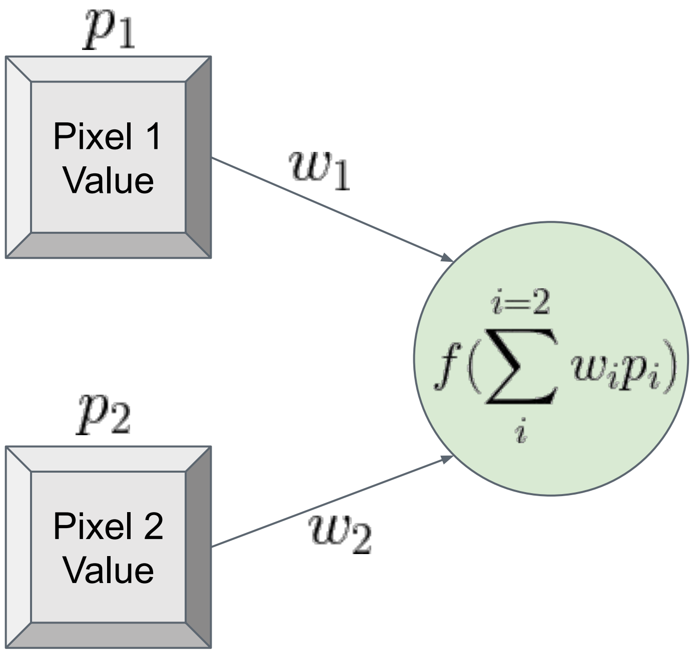
Network Architecture: Data Preprocessing



Sigmoid Activation Function

$$\sigma(x) = \frac{1}{(1 + e^{-x})}$$





Now consider the directionality of the gradient. If all of your inputs into a given neuron are positive, then gradients will all always be positive or negative - no mixing of positive and negative gradients during back propagation.

Features

- Output of function falls between 0 and 1.
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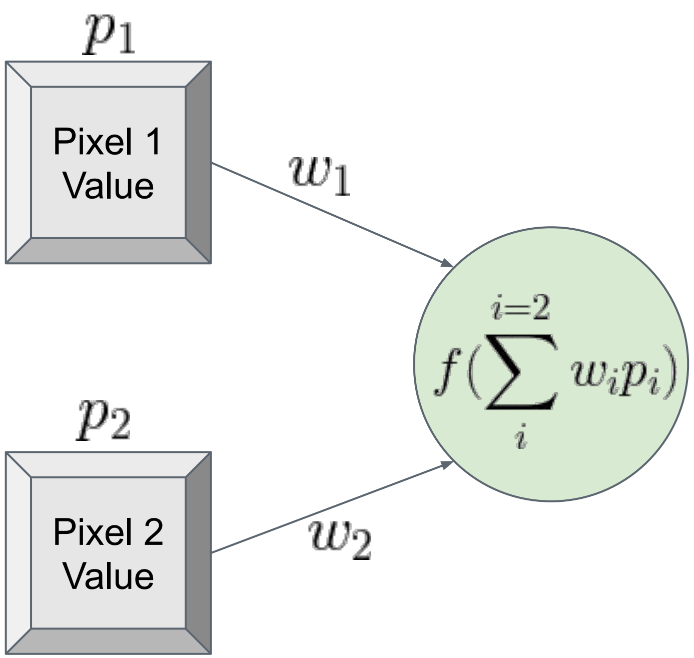
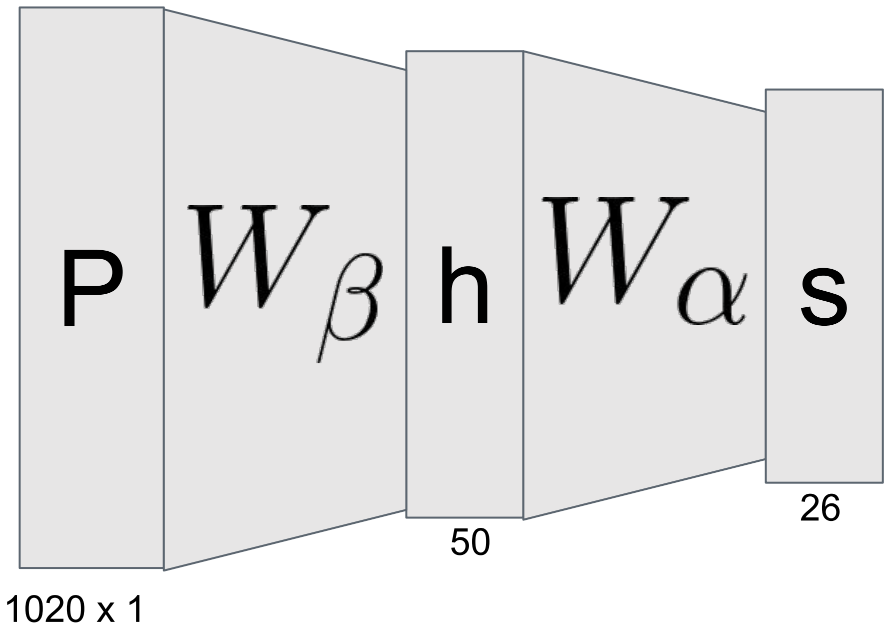
Challenges

- Gradient Decay & Saturation
- Not Zero-Centered & Unidirectional Gradient Solutions

Network Optimization: Weight Initialization





Network Optimization: Weight Initialization

```
W = np.random.randn(3072, 10) * .0001
```

Network Optimization: Weight Initialization

Idea: Big numbers!

W = np.random.randn(3072, 10) * 10

$$tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Network Optimization: Weight Initialization

Idea: ...medium numbers!

(Ok, wait a minute, this is harder than it seemed).

Xavier Initialization

Initial weights should be based on model complexity.

Measurement of complexity: How many inputs and outputs your network has.

Xavier Initialization

Original:

W = np.random.randn(3072, 10) * .0001

Xavier:

W = np.random.randn(3072, 10) / np.sqrt(3072)

Xavier Initialization

<u>**Original:**</u>

W = np.random.randn(3072, 10) * .0001

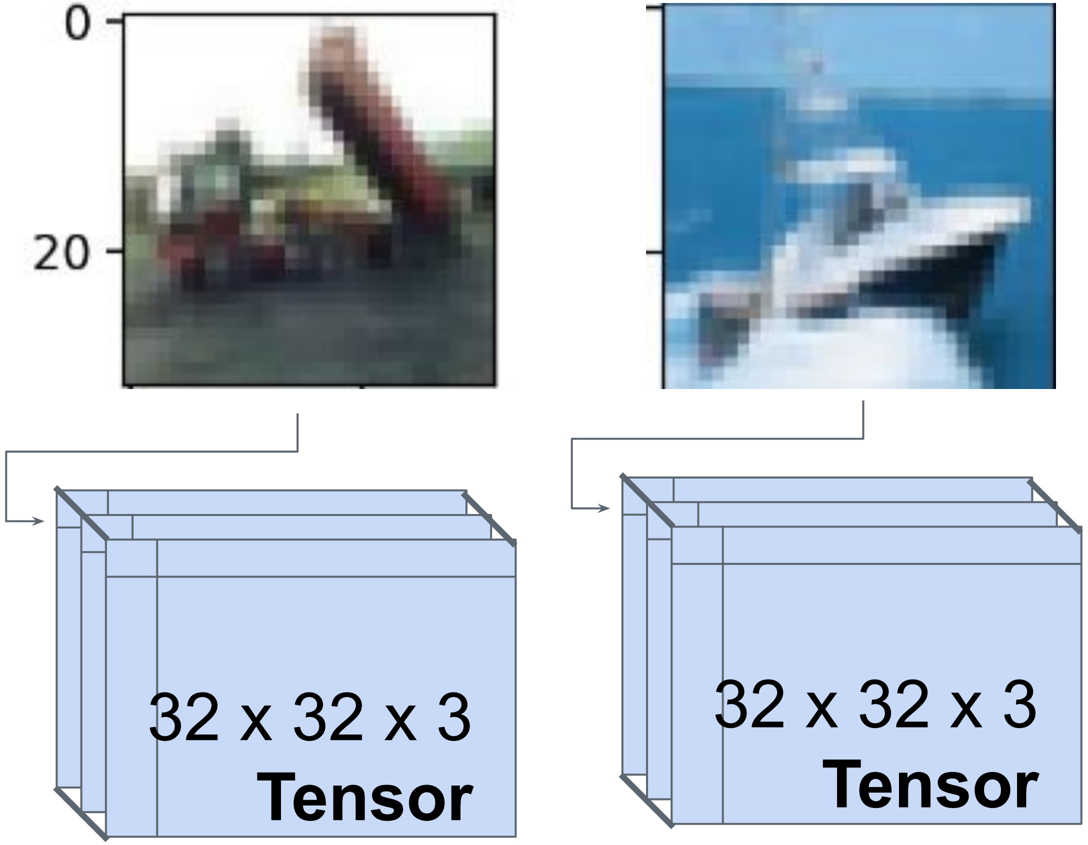
<u>**Xavier:**</u>

W = np.random.randn(3072, 10) / np.sqrt(3072)

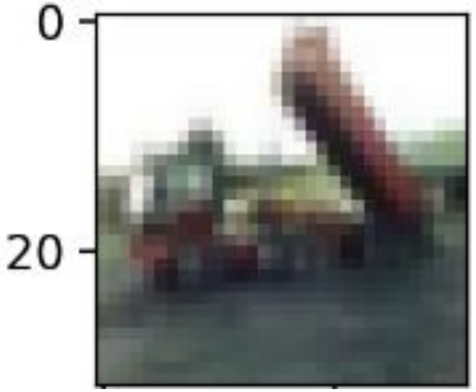
<u>**He:**</u>

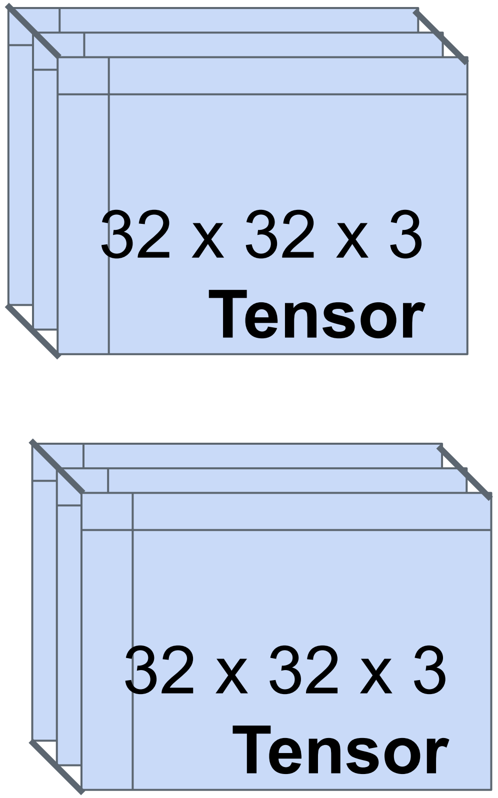
W = np.random.randn(3072, 10) / np.sqrt(3072 / 2)

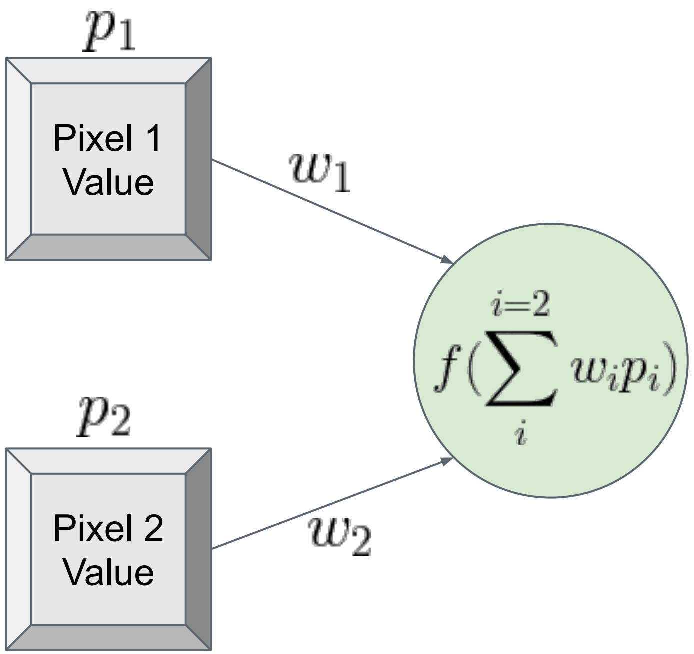
Another Strategy: Batch Normalization

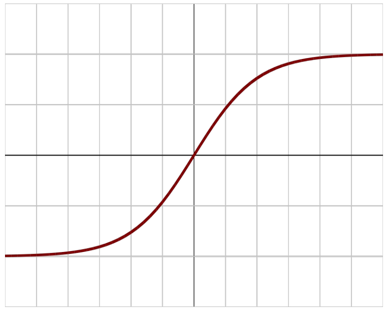


Another Strategy: Batch Normalization



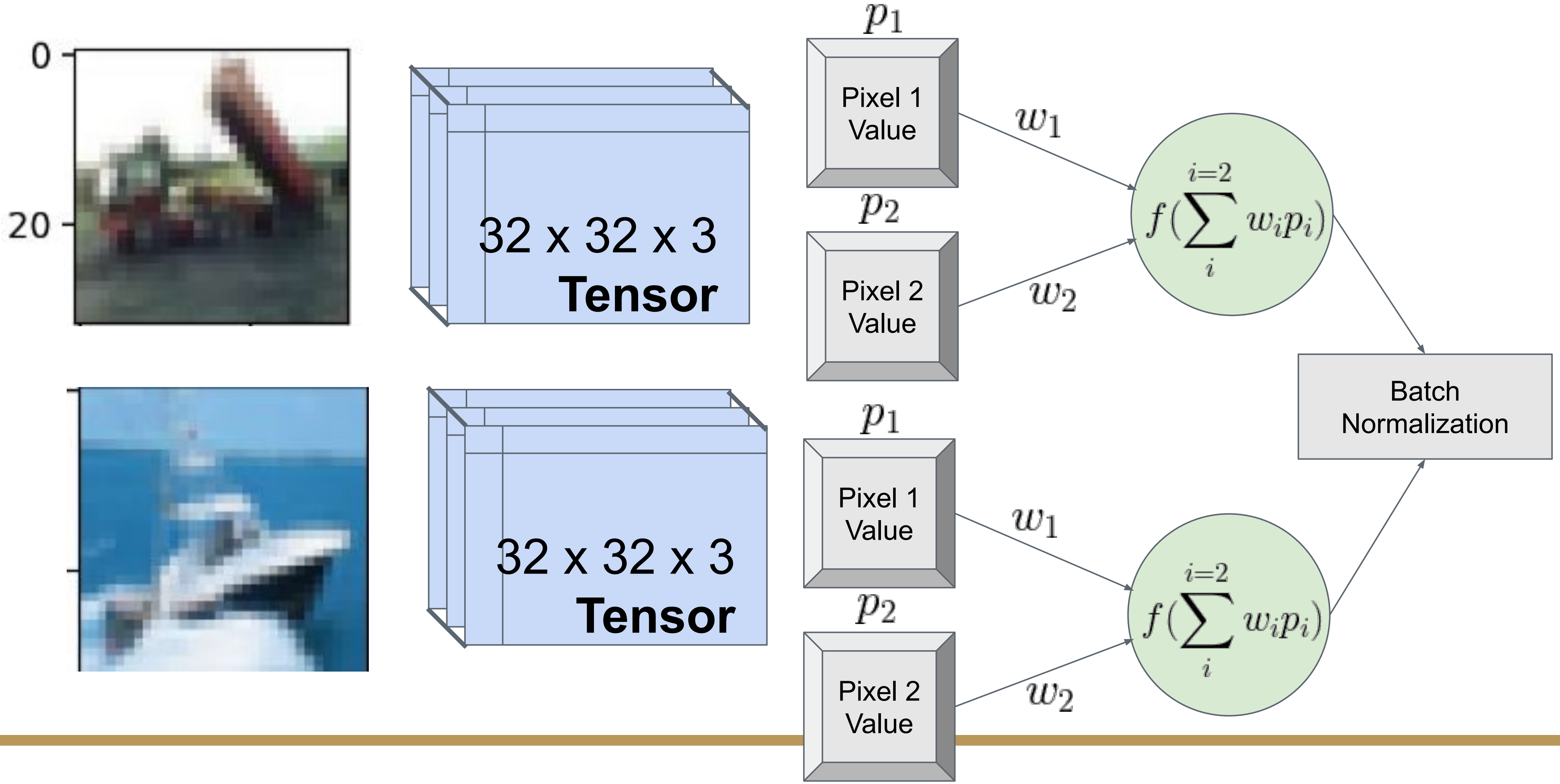




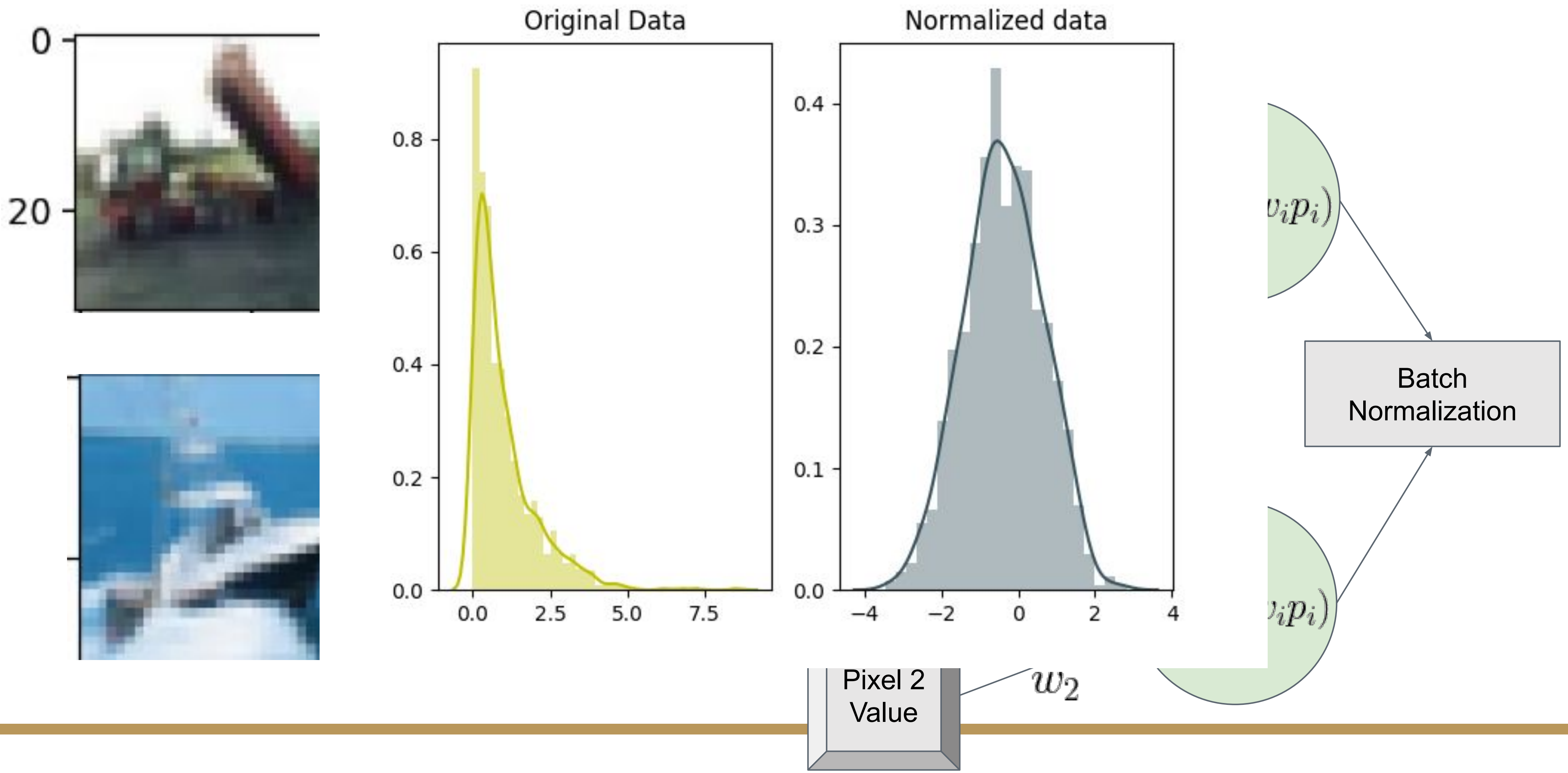


$$tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

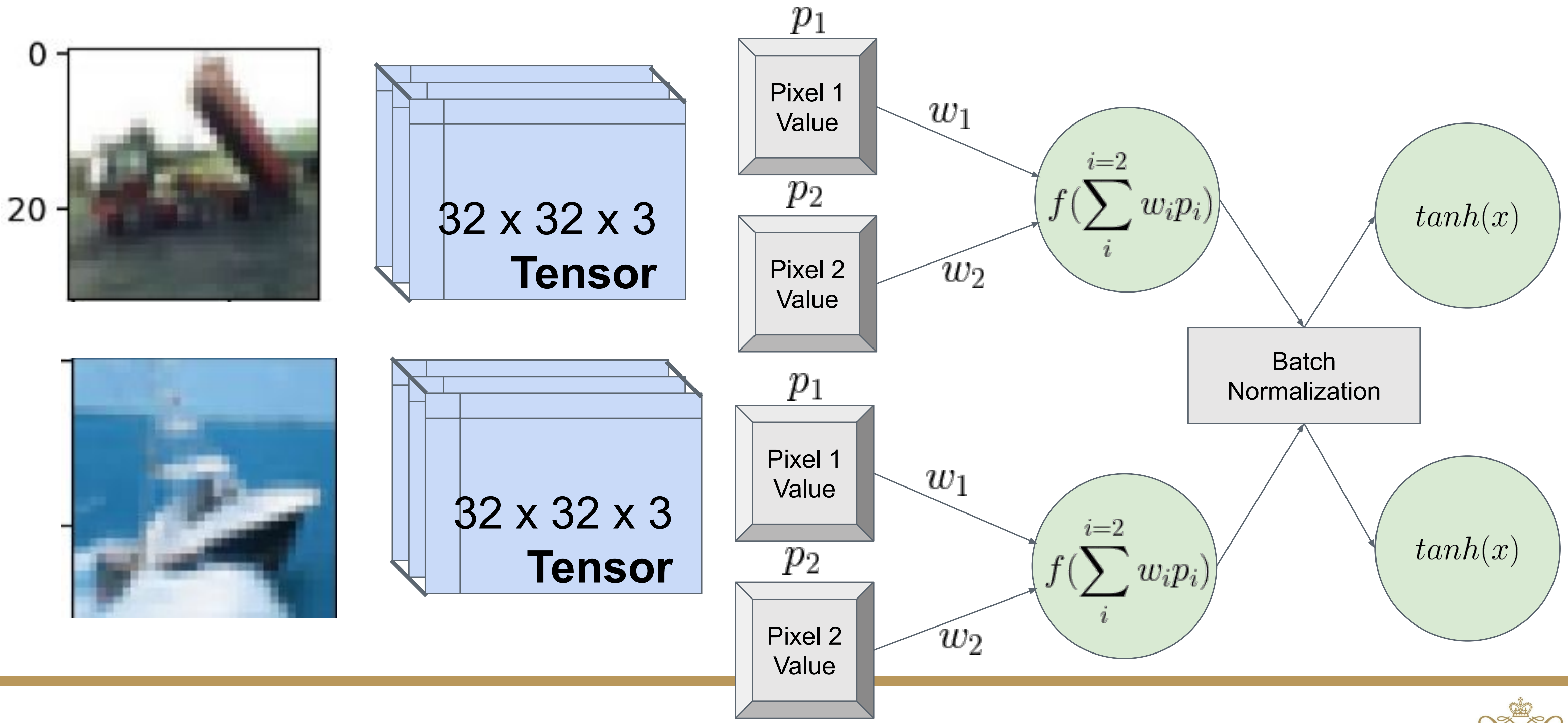
Another Strategy: Batch Normalization



Another Strategy: Batch Normalization



Another Strategy: Batch Normalization



Where we are now

1. Define network architecture (number of hidden layers, inputs, outputs, batch normalizations, activations, etc).
2. Define data preprocessing pipeline (zero-mean standardization).
3. Define weight initializations strategy.