DATA 442: Neural Networks & Deep Learning

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icss.wm.edu/data442/

Summary

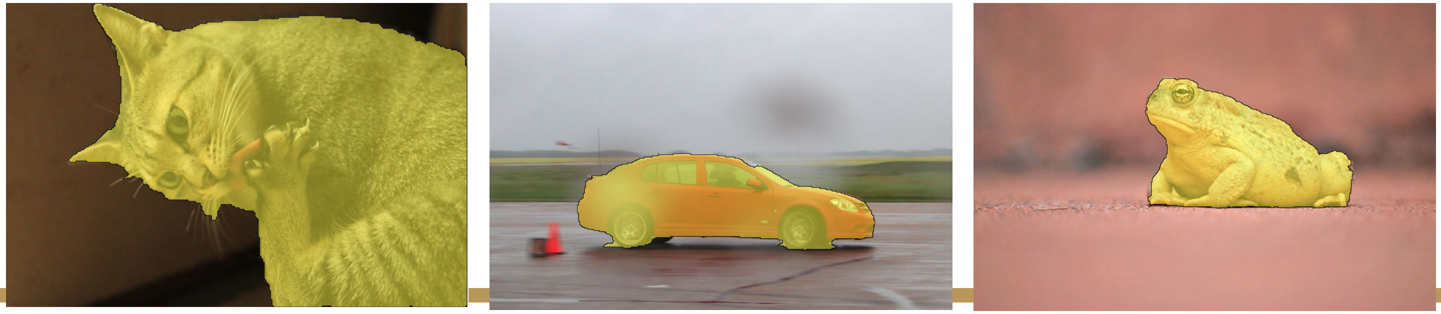
Total Loss=

$$\frac{1}{N}\sum_i^N Loss_i(f(x_i, W), y_i) + \lambda R(W)$$

$$_{i=1}^{N=3}\{(x_i, y_i)\}$$

```
def predict(image, W):
    return(W*image)
```

Cat	3.2	1.3	2.2
Car	5.1	4.9	2.5
Frog	-1.7	2.0	-3.1



$$L_i = -log(\frac{e_k^s}{\sum_{j=1}^{J} e_j^s})$$

Summary

Total Loss$=$

$$\frac{1}{N}\sum_{i}^{N} Loss_i(f(x_i, W), y_i) + \lambda R(W)$$

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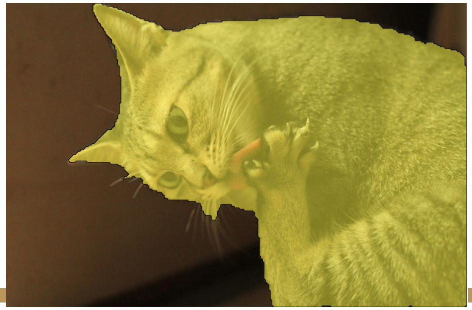
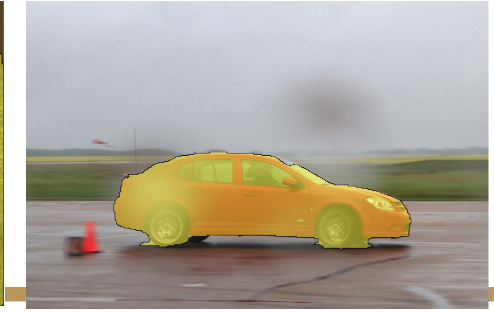
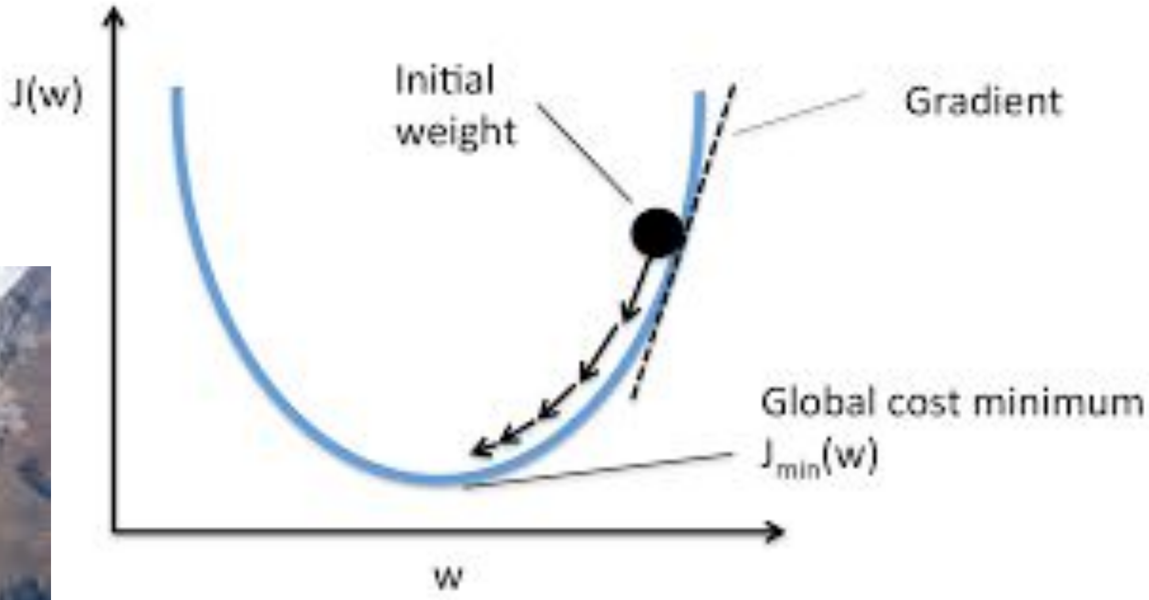
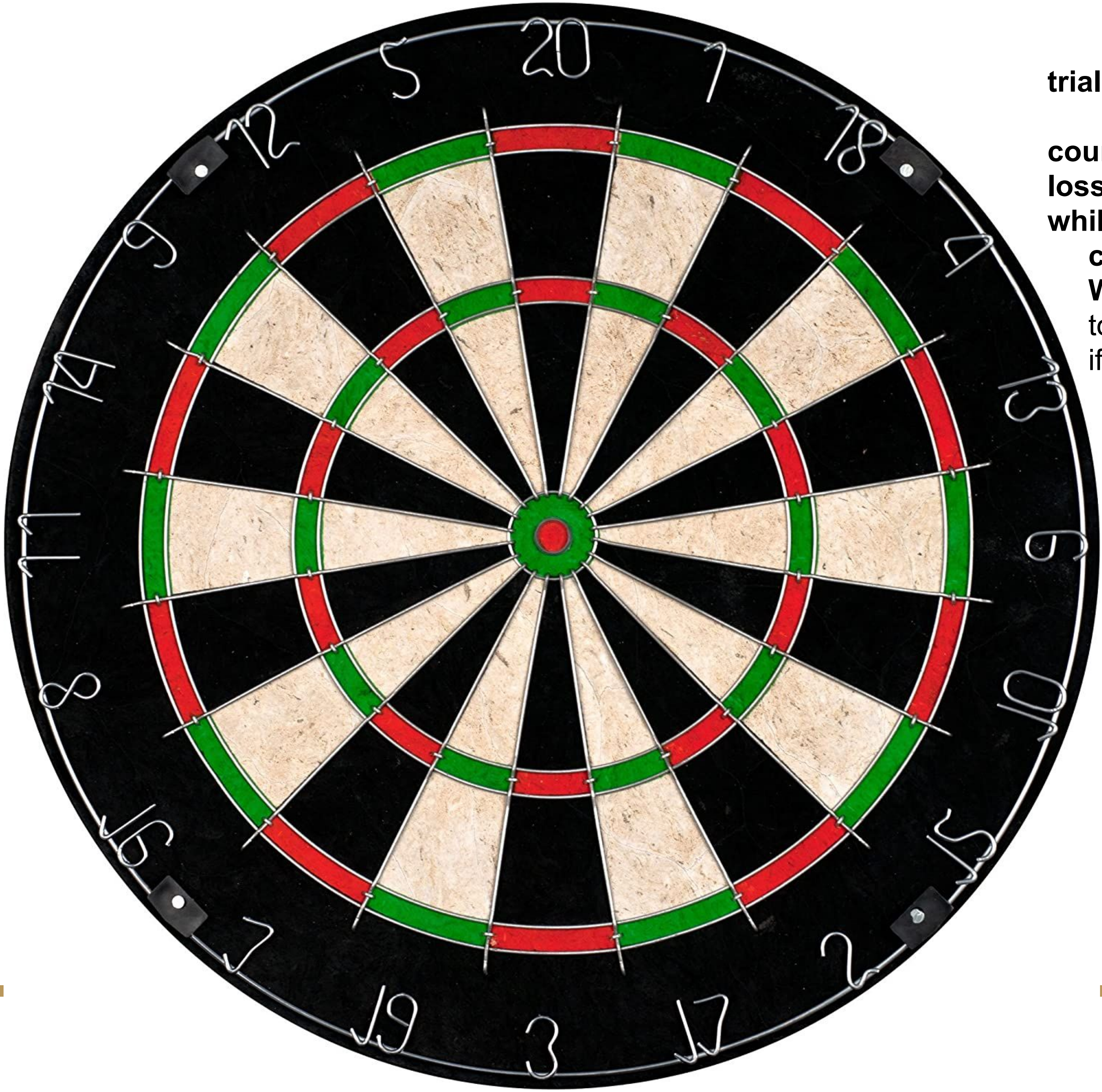




$$L_i = -log(\frac{e_k^s}{\sum_{j=1}^{J} e_j^s})$$

Optimization

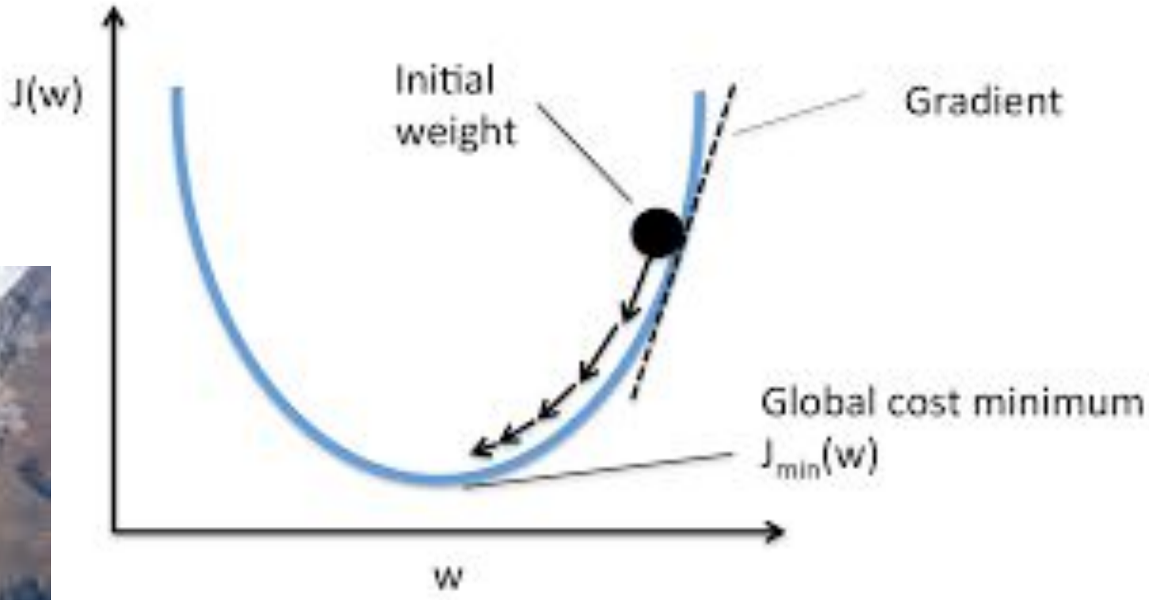




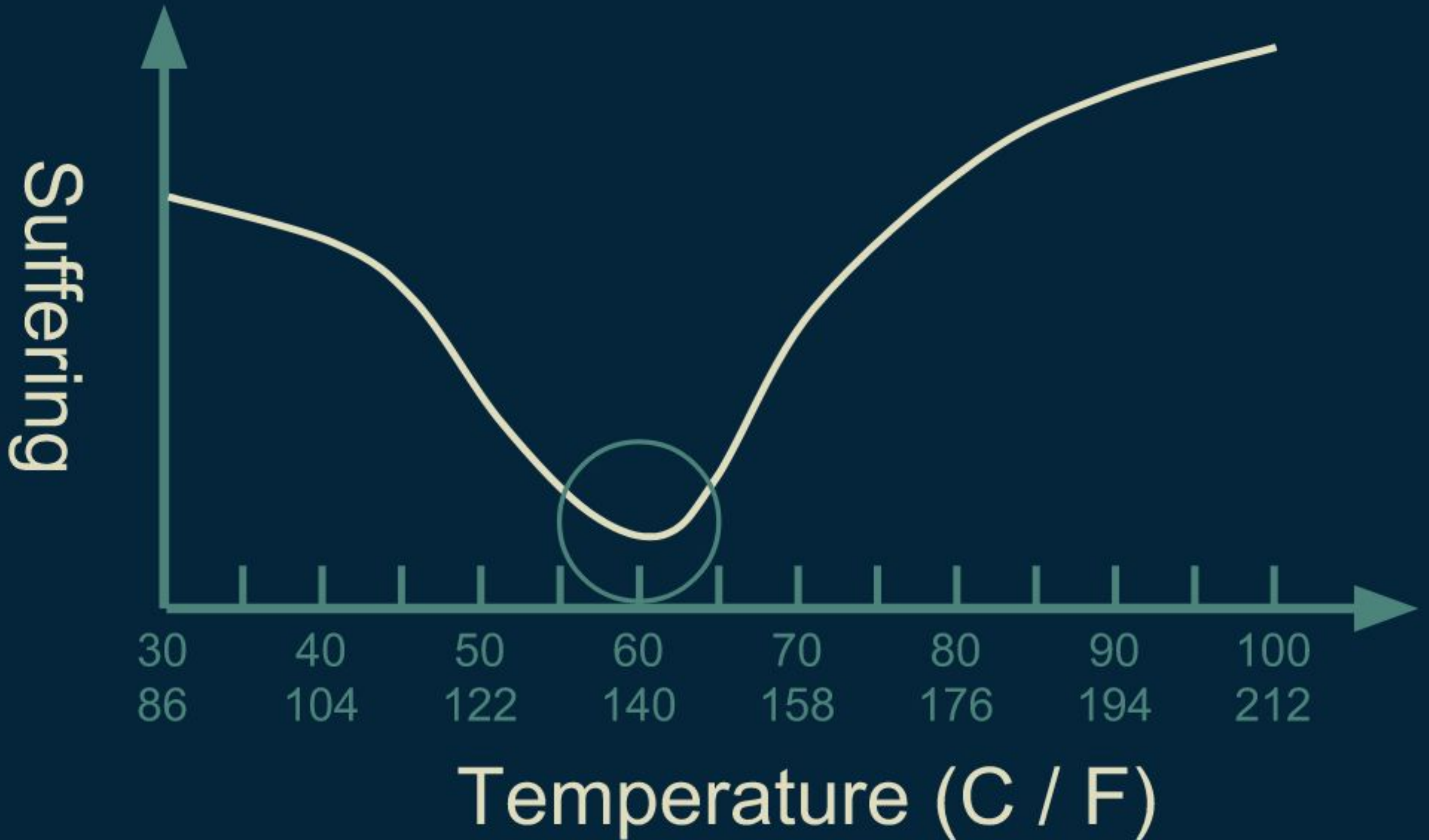

```
trials = 100

count = 0
loss = 99999
while count < trials:
    count = count + 1
    W = np.random.randn(10,3072)
    totalLoss = svmClassifier(X_train, y_train, W, e=1, l=1)
    if totalLoss < loss:
        loss = totalLoss
        finalWeights = W
```

Optimization

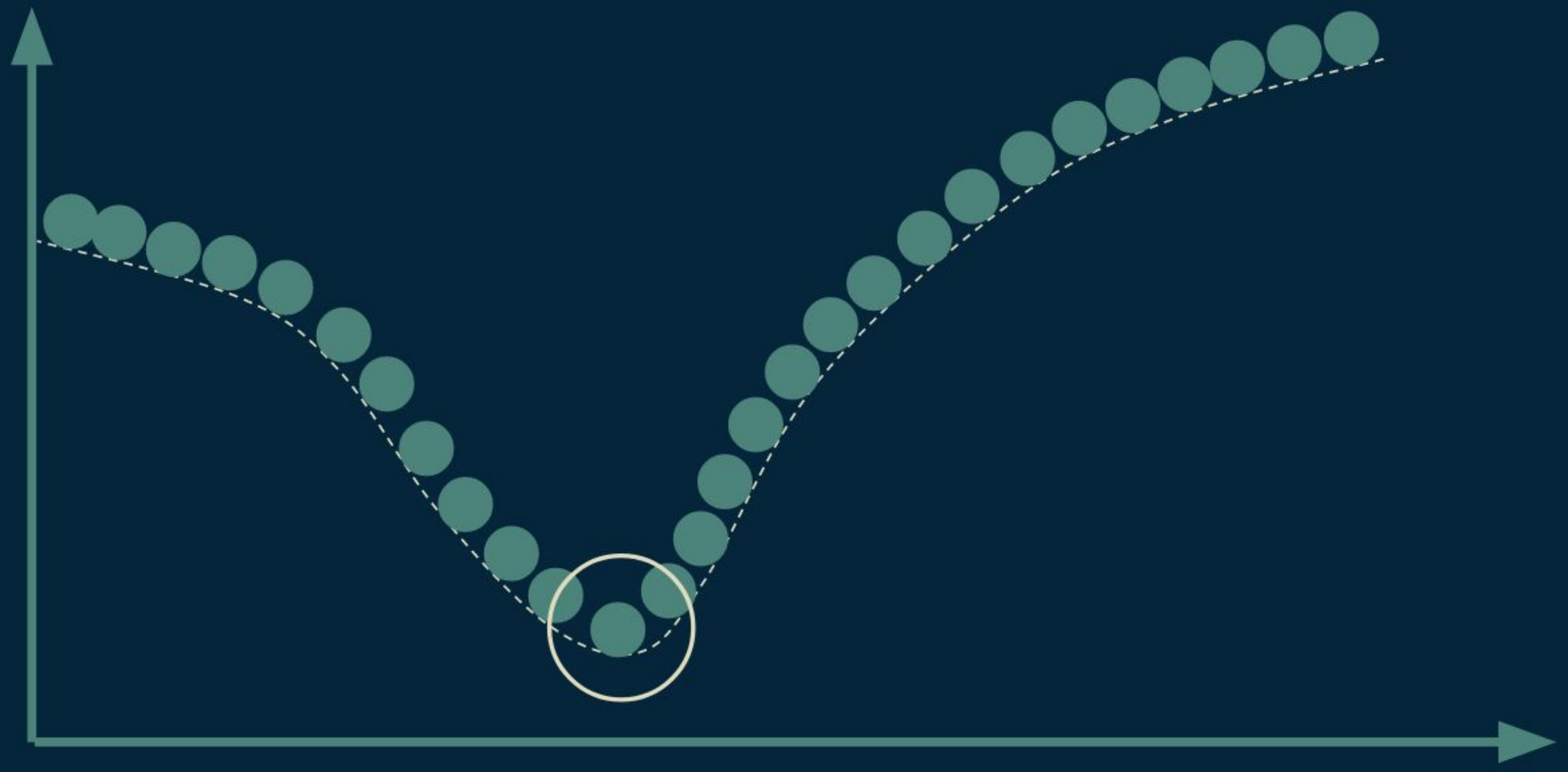


Tea drinking temperature

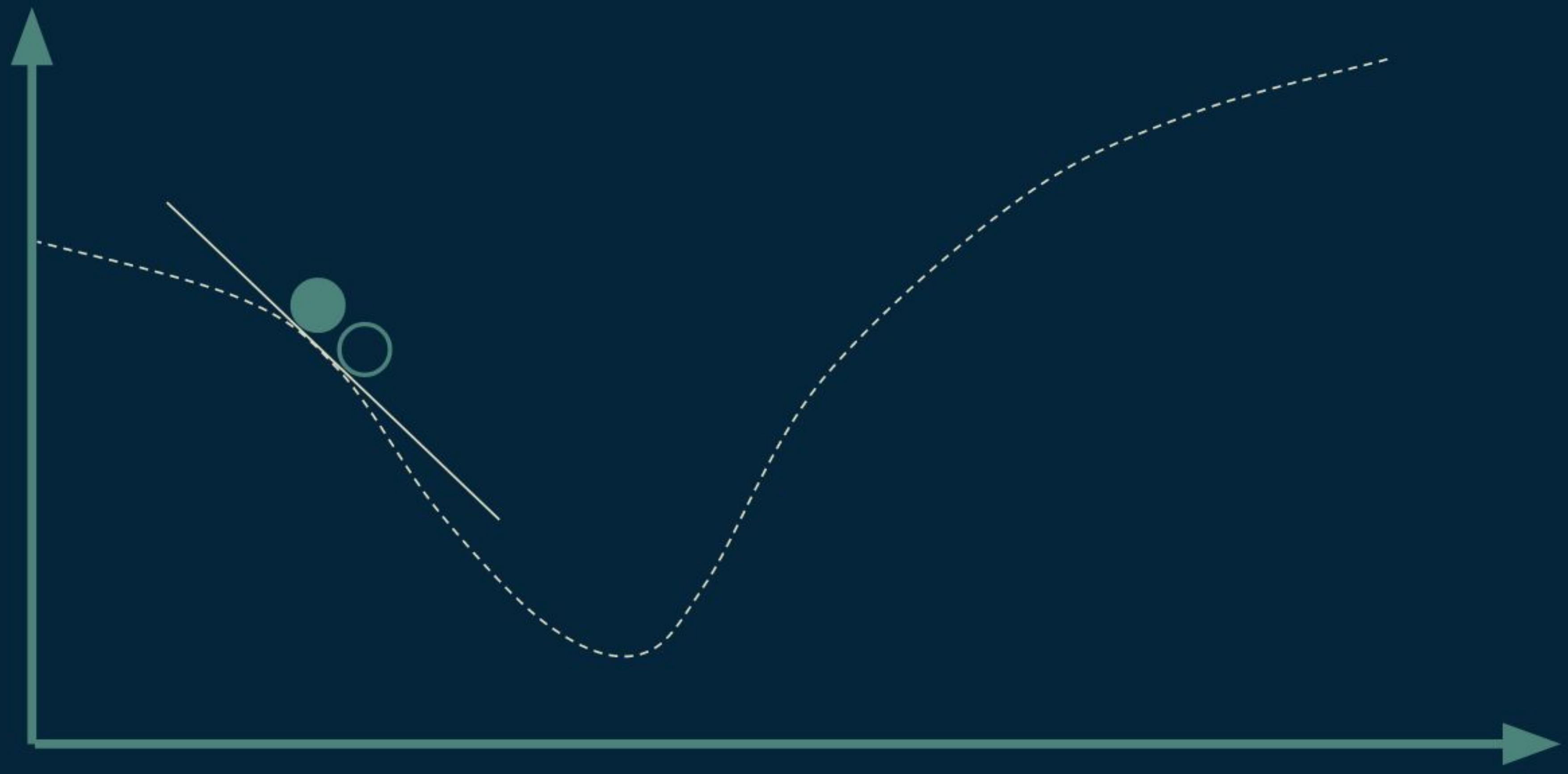


Awesome example from https://brohrer.github.io/how_optimization_works_1.html

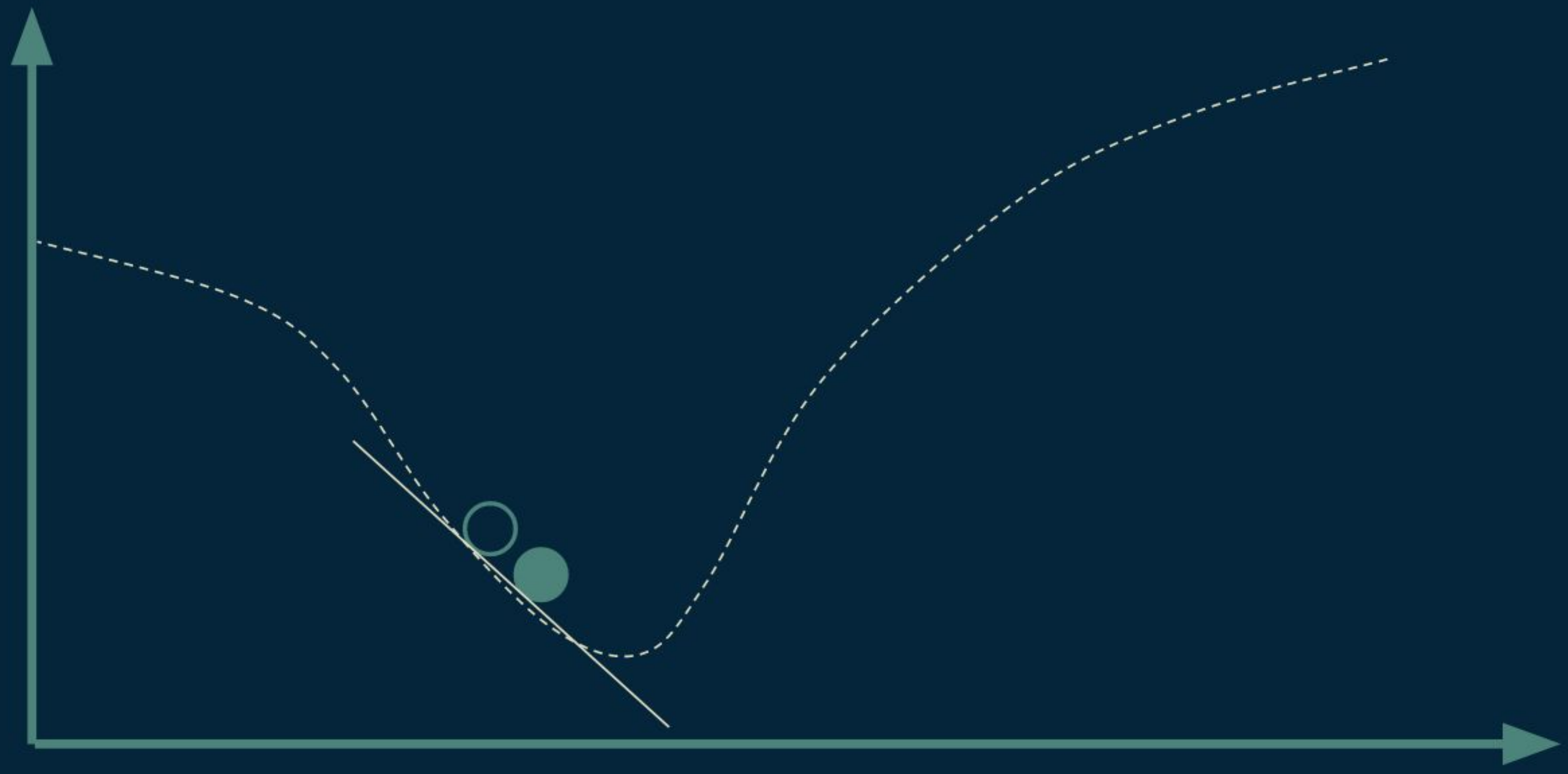
Exhaustive search



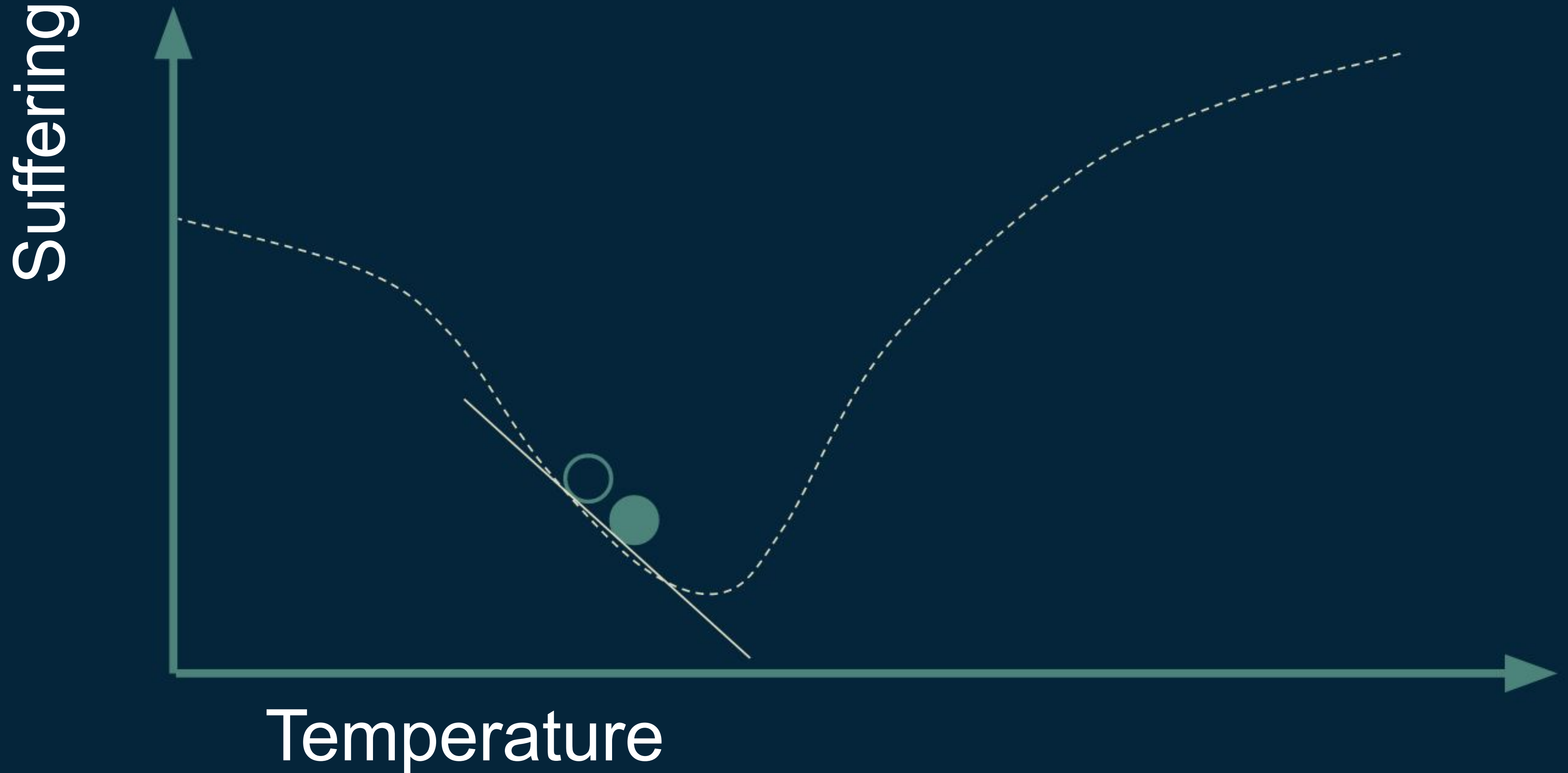
Gradient descent



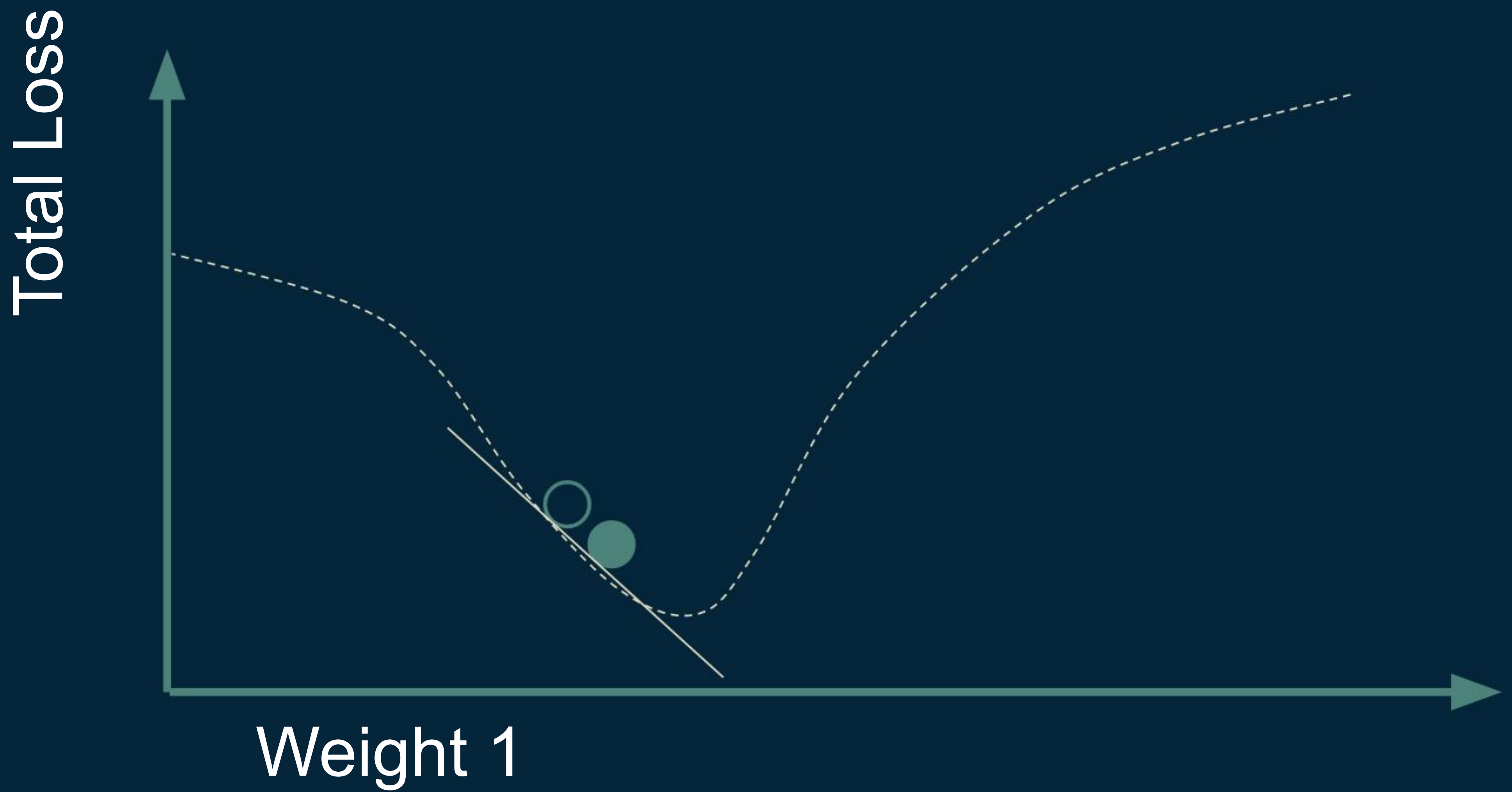
Gradient descent



Gradient descent



Gradient descent



Gradient Descent

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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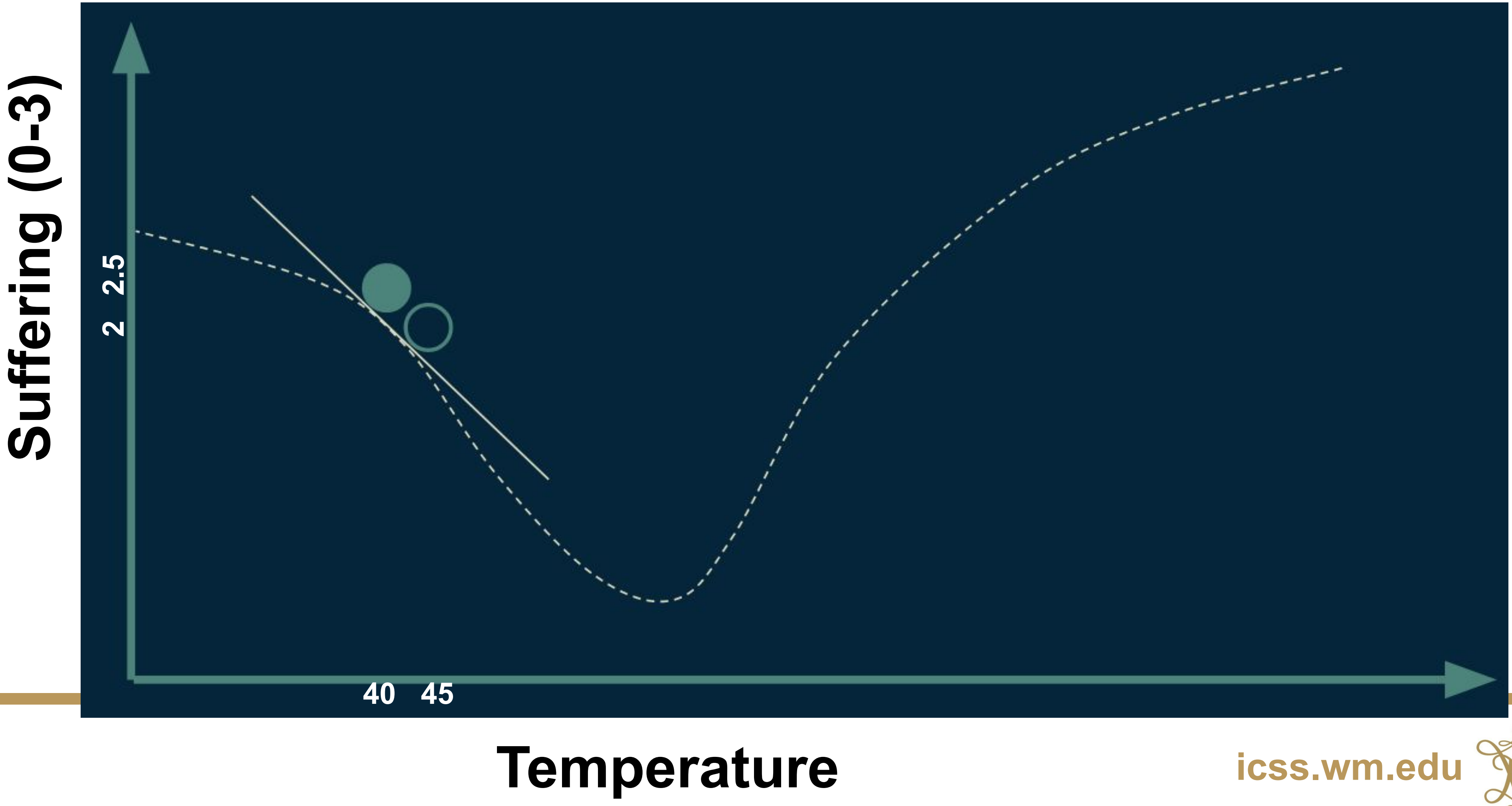
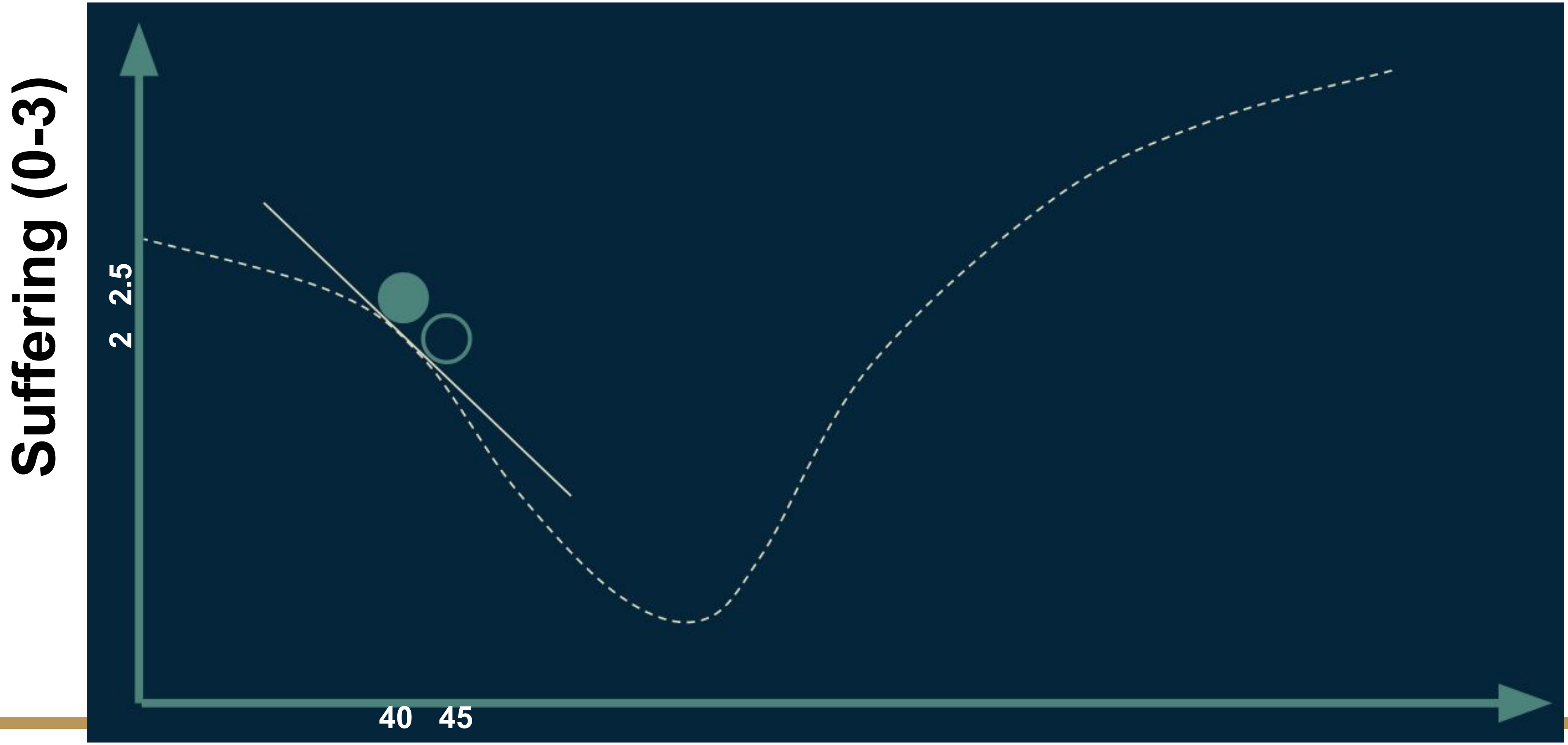


$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(40+5) - f(40)}{5}$$



$$\frac{df(x)}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

In practice we don't just have one variable (temperature). Instead, we have hundreds, thousands, or millions of Weights parameters **(W)**. You can imagine calculating a function similar to the one above for each one of those weights parameters, and getting a resultant vector in which you have one slope for every parameter **W**. This vector is called the **gradient**, and the slopes for each **W** are the partial derivatives.

W = [0.34, -1.11, 0.78, 0.12 … 0.3, 0.77]

Total Loss:
1.25347

Gradient dW: [?, ?, ?, ? … ?, ?]

W = **[0.34, -1.11, 0.78, 0.12 … 0.3, 0.77]**

Total Loss: 1.25347

h = .0001

W+h: [<u>0.34 + 0.0001</u>, -1.11, 0.78, 0.12 … 0.3, 0.77]

Gradient dW: **[?, ?, ?, ? … ?, ?]**

W = **[0.34, -1.11, 0.78, 0.12 … 0.3, 0.77]**

Total Loss:
1.25347

h = .0001

W+h: [<u>0.34 + 0.0001</u>, -1.11, 0.78, 0.12 … 0.3, 0.77]

Total Loss:
1.25322

Gradient
dW: **[?, ?, ?, ? … ?, ?]**

W = **[0.34, -1.11, 0.78, 0.12 … 0.3, 0.77]**

Total Loss: 1.25347

W+h: [0.34 + 0.0001, -1.11, 0.78, 0.12 … 0.3, 0.77]

Total Loss: 1.25322

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(1.25322 - 1.25347) / .0001 = -2.5

Gradient dW: **[-2.5, ?, ?, ? … ?, ?]**

W = [0.34, -1.11, 0.78, 0.12 … 0.3, 0.77]

Total Loss:
1.25347

W+h: [0.34, -1.11+.0001, 0.78, 0.12 … 0.3, 0.77]

Total Loss:
1.25353

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(1.25353 - 1.25347) / .0001 = 0.6

Gradient dW: [-2.5, 0.6, ?, ? … ?, ?]

W = [0.34, -1.11, 0.78, 0.12 … 0.3, 0.77]

W+h: [0.34, -1.11+.0001, 0.78, 0.12 … 0.3, 0.77]

Gradient

dW: [-2.5, 0.6, 4.3, 0.5 … 0, 0.3]

Analytic Gradient

W = [0.34, -1.11, 0.78, 0.12 … 0.3, 0.77]

$$dw = f(X, W)$$

$$\nabla f(X, W) = [\ldots]$$

Gradient

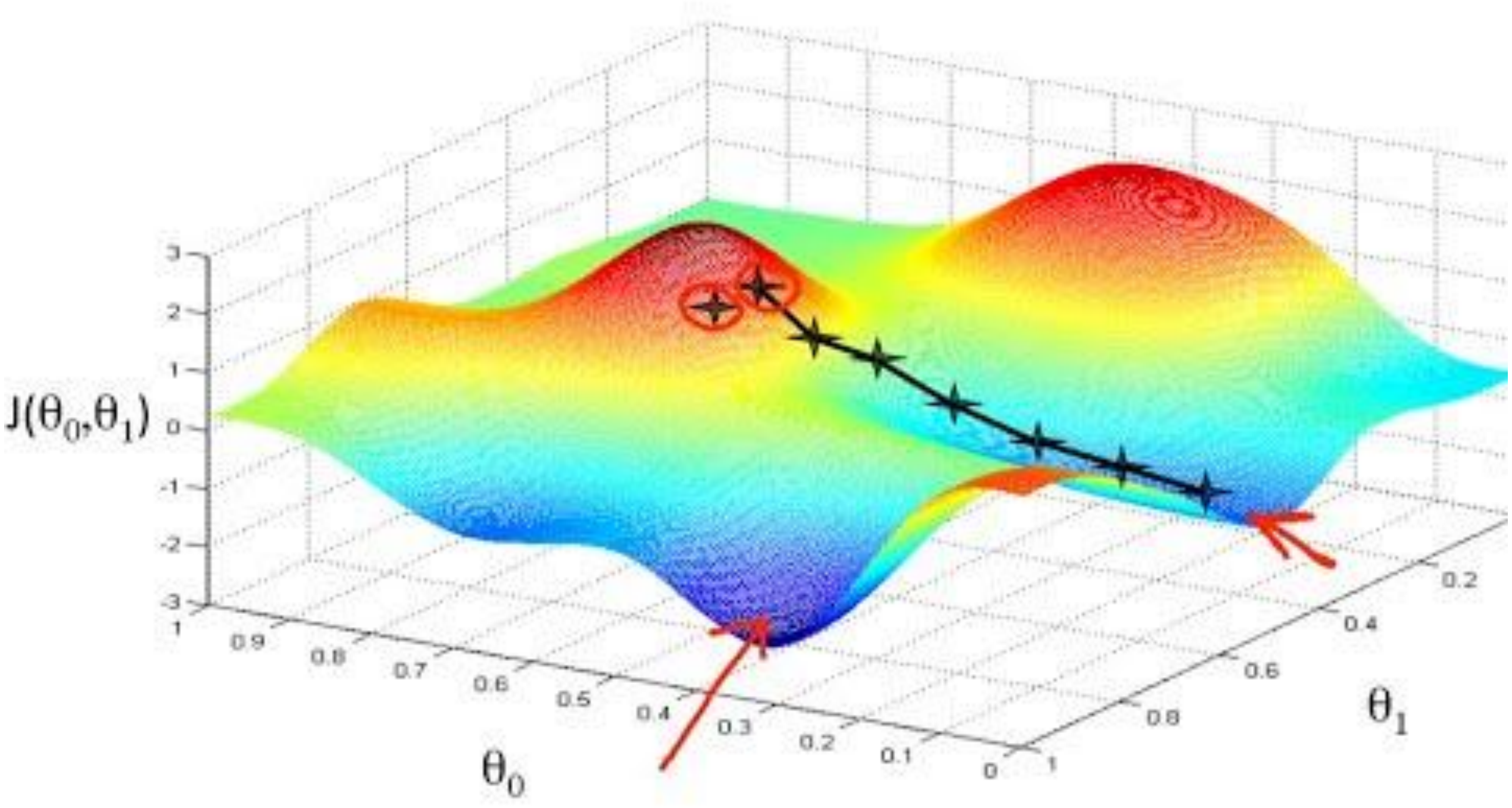
dW: [-2.5, 0.6, 4.3, 0.5 … 0, 0.3]

Gradient Descent in Code

```
maxIterations = 1000
count = 0

while count < maxIterations:
    count = count + 1
    W_gradient_dW = calculateGradient(lossFunction, X, W)
    W = W + -1 * (stepSize * W_gradient_dW)
```

[redacted]



[redacted]

Batch Sizes & Stochastic Gradient Descent

```
maxIterations = 1000
count = 0

while count < maxIterations:
    count = count + 1
    W_gradient_dW = calculateGradient(lossFunction, X, W)
    W = W + -1 * (stepSize * W_gradient_dW)
```

Can be VERY slow for large training datasets.

Batch Sizes & Stochastic Gradient Descent

```
while count < maxIterations:
    count = count + 1
    X_sample = X.sample(n=256)
    W_gradient_dW = calculateGradient(lossFunction, X_sample, W)
    W = W + -1 * (stepSize * W_gradient_dW)
```

Batch Size (can be anything, normally 8,16,32,64 depending on memory and weights.

Recap

- What is optimization?
- How does it interrelate with the loss function?
- How can we solve for W using random guessing or an exhaustive search?
- What is gradient descent and stochastic gradient descent, and how does SGD interrelate with batch size?